PreAP Calculus Unit 5 Homework Name______________________________

REVIEW 5 Day 1

Calculator NOT Permitted

FRQ 1: The table below shows function values for a rational function, $G(x)$. The equation of $G(x)$ is such that $(x + 2)$ and $(x - 1)$ are the only factors in the denominator of the function.

x	–1000	–2.001	–2	–1.999	0	0.999	1	1.001	1000
$G(x)$	0.998	0.333	Undefined	0.333	–1	–1999	Undefined	2001	1.002

a. Does either factor in the denominator also exist in the numerator? If so, which factor? Give a reason for your answer.

$\lim_{x \to -2^-} G(x) = \frac{1}{3}$

+1 $\lim_{x \to -2^+} G(x) = \frac{1}{3}$

$G(-2) \neq \frac{1}{3}$

$\therefore$ $G(x)$ has point discontinuity at $x = -2$

$\therefore$ $(x+2)$ is a factor in both the numerator and denominator. +1

b. Does either factor of the denominator not exist in the numerator? If so, which factor? Give a reason for your answer.

$\lim_{x \to 1^-} G(x) = -\infty$ +1

$\lim_{x \to 1^+} G(x) = \infty$

$\therefore$ $G(x)$ has a vertical asymptote at $x = 1$ +1

$\therefore$ $(x-1)$ is a factor of the denominator only.

c. Based on the end behavior, where does $G(x)$ have a horizontal asymptote? Give a reason for your answer.

$\lim_{x \to -\infty} G(x) = 1$

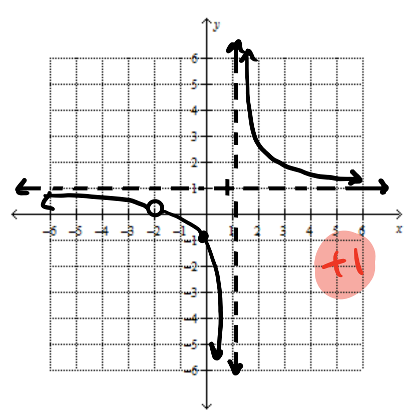
+1 $\lim_{x \to \infty} G(x) = 1$

$\therefore$ $G(x)$ has a horizontal asymptote at $y = 1$ +1

d. Sketch a possible graph of the function $G(x)$. Then, state the domain and range of $G(x)$.

Domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ +1

Range: $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, 1) \cup (1, \infty)$ +1



Calculator NOT Permitted

FRQ 2: Consider the rational functions $f(x) = \frac{2}{x-3}$ and $g(x) = \frac{x-2}{x^2-9}$ to answer the following questions.

$\rightarrow (x-3)(x+3)$

a. At what value(s) of x will the graphs of $f(x)$ and $g(x)$ have discontinuities? Explain your reasoning.

The graphs of f(x) and g(x) will have discontinuities when the denominators are equal to zero. +1

$\therefore$ $f(x)$ is undefined at $x = 3$

$\therefore$ $g(x)$ is undefined at $x = -3$ and $x = 3$ +1

b. If $h(x) = f(x) \cdot g(x)$, find an equation for h in standard form and then determine at what value of y will the graph of h have a horizontal asymptote? If no such value exists, state so. Give a reason for your answer.

$$h(x) = \frac{2}{x-3} \cdot \frac{x-2}{x^2-9}$$

In h(x), degree of the numerator is less than the denominator. +1

$$h(x) = \frac{2x-4}{x^3-9x-3x^2+27}$$

$\therefore$ h(x) has a horizontal asymptote at $y=0$. +1

+1 $$h(x) = \frac{2x-4}{x^3-3x^2-9x+27}$$

c. On what intervals will $f(x) \geq g(x)$? Show the complete algebraic and sign analysis that leads to your answer.

$$\frac{2}{x-3} \geq \frac{x-2}{x^2-9}$$

$$\frac{(x+3)}{(x+3)}\frac{2}{x-3} - \frac{x-2}{(x-3)(x+3)} \geq 0 \quad +1$$

$$\frac{2x+6}{(x+3)(x-3)} - \frac{x-2}{(x-3)(x+3)} \geq 0$$

$$\frac{x+8}{(x-3)(x+3)} \geq 0$$

+1

$\frac{(-)}{(-)(-)}$ $\quad \frac{+}{(-)(-)}$ $\quad \frac{+}{(-)(+)}$ $\quad \frac{+}{(+)(+)}$

$x=-10$ $\quad x=-5$ $\quad x=0$ $\quad x=10$

-8 $\quad \cancel{-3}$ $\quad \cancel{3}$ +1

NEG $\quad$ POS $\quad$ NEG $\quad$ POS

$f(x) \geq g(x)$ on $[-8,-3) \cup (3,\infty)$ +1

 Name____________________

Calculator NOT Permitted

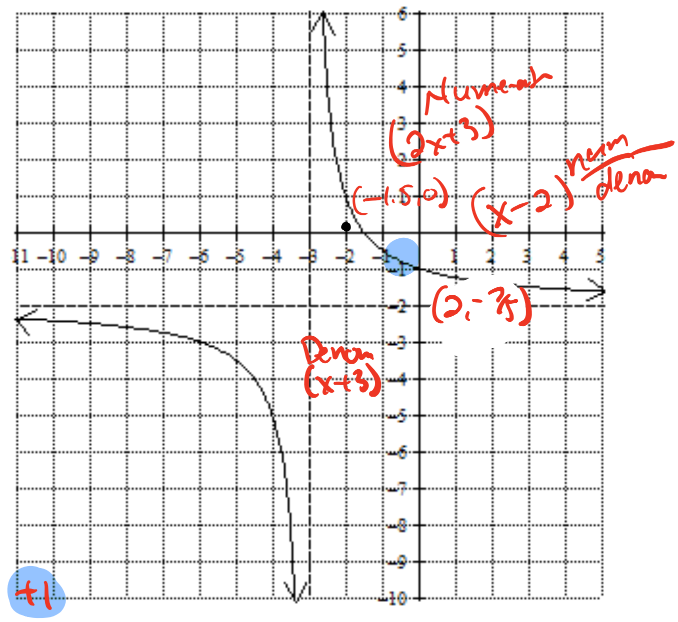
$(x+1.5)$ → $(2x+3)$ N

FRQ 3: The graph of the rational function, $h(x)$, pictured to the right is such that $h(-1.5) = 0$ and has a hole at the point $\left(2, -\frac{7}{5}\right)$. $(x-2)$ N/D

a. State the domain and range of $h(x)$.

Domain $= (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ +1

Range $= (-\infty, -2) \cup (-2, -7/5) \cup (-7/5, \infty)$ +1



b. What factor(s) is/are guaranteed to be in the denominator of the equation of $h(x)$? Give a reason for your answer.

FACTORS IN THE Denominator Cause discontinuities.
$h(x)$'s graph has discontinuities at $x = -3$ and $x = 2$ +1

∴ $h(x)$ has factors of $(x+3)$ and $(x-2)$ in the denominator +1

c. For what value(s) of x is $h(x) \le 0$? Give a graphical reason for your answer.

If $h(x) \le 0$, then the graph of $h(x)$ is on or below the x-axis. +1

∴ $h(x) \le 0$ on $(-\infty, -3) \cup [-1.5, 2) \cup (2, \infty)$ +1

d. In standard form, find an equation for $h(x)$. Give two graphical and algebraic connections that confirm that your standard form equation is correct.

$$h(x) = \frac{-(2x+3)(x-2)}{(x+3)(x-2)}$$

$$h(x) = \frac{-(2x^2 - x - 6)}{x^2 + x - 6}$$

+1 $$h(x) = \frac{-2x^2 + x + 6}{x^2 + x - 6}$$

- In $h(x)$ the ratio of the constants is $\frac{6}{-6} = -1$, which is the y-int. +1
- $h(x)$'s graph has a hole at $x = 2$ and $h(x)$ has $(x-2)$ as factors of the numerator & denominator +1
- The graph of $h(x)$ has a HA @ $y = -2$ and $h(x)$ has same degree in numerator and denominator with ratio of lead coefficients $\frac{-2}{1} = -2$
- The graph of $h(x)$ has a VA @ $x = -3$ and $h(x)$ has a non-canceling factor of $(x+3)$ in the denominator.
- The graph of $h(x)$ has a zero at $x = -1.5$ and $h(x)$ has a non canceling factor of $(2x+3)$ in the numerator

Multiple Choice – Calculator Permitted

zero @ $x = 3/2$

y-int $= \frac{3 \cdot 3}{3(-1)}$

1. If $P(x) = \frac{(3-2x)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)}$, then which of the following statements is/are true?

VA @ $x = 1$ HA @ $y = \frac{-2}{1}$

Hole @ $x = (-3, \frac{9}{-4})$

I. The graph of $P(x)$ has a horizontal asymptote at $y = -2$.

II. The graph of $P(x)$ has a point discontinuity at $\left(-3, -\frac{9}{4}\right)$.

III. The graph of $P(x)$ has a y – intercept at (0, −3).

A. I and II only B. I and III only C. II and III only

D. I only E. I, II, and III

$2x^2 - 3x - 9$

$2x^2 - 6x + 3x - 9$

$2x(x-3) + 3(x-3)$

$(x-3)(2x+3)$

VA

2. Identify the x – value(s) of any non-removable discontinuity in the function $f(x) = \frac{2x^2-3x-9}{x^2-9}$. $= \frac{\cancel{(x-3)}(2x+3)}{\cancel{(x-3)}(x+3)}$

VA @ $x = -3$

Hole VA

I. $x = 3$ II. $x = -3$ III. $x = -\frac{3}{2}$

A. I only B. II only C. I and II only

D. II and III only E. I, II and III

3. A rational function has a vertical asymptote at $x = -1$, a horizontal asymptotes at $y = 7$ and $y = 9$ and a hole in the graph at the point (4, −5). What are the domain and range of the rational function?

D: $\mathbb{R}$, $x \neq -1, 4$ R: $\mathbb{R}$, $y \neq -5, 7, 9$

A. Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

B. Domain: $(-\infty, -1) \cup (-1, \infty)$ Range: $(-\infty, \infty)$

C. Domain: $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$ Range: $(-\infty, 7) \cup (7, 9) \cup (9, \infty)$

D. Domain: $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$ Range: $(-\infty, -5) \cup (-5, 7) \cup (7, 9) \cup (9, \infty)$

E. Domain: $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$ Range: $(-\infty, 7) \cup (7, \infty)$

Hole @ $x=3$

4. What does the graph of $f(x) = \frac{x^2-3x}{x^2+2x-15}$ look like at the value x = 3. $\frac{x\cancel{(x-3)}}{\cancel{(x-3)}(x+5)}$

   A. There is a vertical asymptote at *x* = 3.
   B. There is a point discontinuity at *x* = 3.
   C. There is a jump in the graph at *x* = 3.
   D. The graph is continuous at *x* = 3.
   E. The graph passes through and includes the point $\left(3, \frac{3}{8}\right)$.

5. Which of the following statements is/are true about the rational function $f(x) = \frac{x(x-10)\cancel{(x+7)}}{x\cancel{(x+7)}}$?

   I. The graph of *f*(*x*) has a hole in it at the point $(-7, -17)$. ✓ Hole @ $\left(-7, \frac{-7(-17)}{-7}\right)$
   II. The graph of *f*(*x*) has a vertical asymptote at *x* = –7. ✗
   III. The graph of *f*(*x*) will cross the *x* – axis at *x* = 0, x = 10 and *x* = –7. ✗

   A. I, II and III
   B. I and II only
   C. II only
   D. I and III only
   E. I only

6. Which of the following rational functions have a point discontinuity at *x* = 5 and a horizontal asymptote that is on the *x* – axis?

   $(x-5)\ n/D$ HA: $n<d$

   $n=d$

   I. $f(x) = \frac{x^2-2x-15}{3x^2-16x+5}$ ✗

   $\frac{\cancel{x-5}}{\cancel{(x-5)}(2x+3)}$ ✓

   II. $g(x) = \frac{x-5}{2x^2-7x-15}$

   $2x^2-10x+3x-15$

   $2x(x-5)+3(x-5)$

   $(x-5)(2x+3)$

   III. $h(x) = \frac{2x-6}{x^2-8x+15} = \frac{2(x-3)}{(x-3)(x-5)}$ ✗

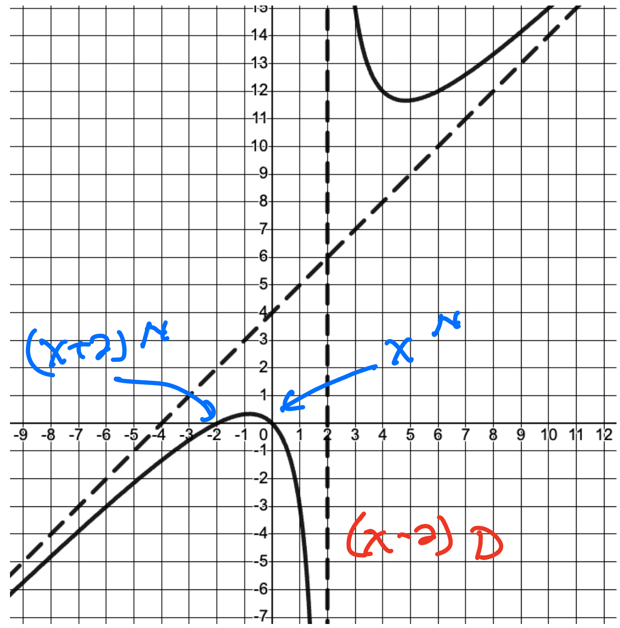
   A. I only
   B. I and II only
   C. II only
   D. III only
   E. II and III only
   F. I and III only

7. Which of the following statements is/are true about the function, $g(x)$, whose graph is pictured to the right?

I. The factors $(x + 2)$ and x are guaranteed to be in the numerator of the equation of $g(x)$.

II. The leading coefficient of the numerator of the equation of $g(x)$ is 3.

III. The factor $(x - 2)$ is guaranteed to be in the denominator of the equation of $g(x)$.



A. I only

B. I and III only

C. I and II only

D. III only

E. I, II, and III

$$g(x) = \frac{x(x+2)}{(x-2)}$$

SA: $y=x$

REVIEW 5 Day 2

Calculator NOT Permitted

FRQ 5: The rational function $F(x) = \frac{(ax-3)(x-2)}{2(x+3)(x-2)}$ is such that *y* = −2 is a horizontal asymptote.

a. Why does the provided equation support the fact that there is a horizontal asymptote of *F*(*x*) that is not on the *x* – axis?

The degree of the numerator and denominator are equal. +1

∴ F(x) has a horizontal Asymptote that is not on x-axis. +1

b. Find the correct value of *a*. Show your work and justify your thinking.

The degree of the numerator and denominator are equal. +1

∴ The horizontal asymptote is the ratio of the lead coefficients

$\frac{a}{2} = -2$ +1

$a = -4$

c. Does the graph of *F*(*x*) have any vertical asymptotes? Why or why not? If any vertical asymptotes exist, what is/are the equations?

F(x) has a non canceling factor of (x+3) in the denominator. +1

∴ F(x) has a vertical asymptote at x = -3 +1

d. Does the graph of *F*(*x*) have any holes in it? Why or why not? If any holes exist, what are the coordinates of the holes?

F(x)'s numerator and denominator share a factor of (x-2). +1

∴ F(x) has a hole at x = 2 +1

$F(x) = \frac{-4x-3}{2(x+3)}$

$F(2) = \frac{-4(2)-3}{2(2+3)} = \frac{-8-3}{2(5)} = \frac{-11}{10}$

HOLE $(2, \frac{-11}{10})$ +1

Multiple Choice – Calculator NOT Permitted

$(x-3)$ N

8. If it is known that $p(3) = 0$, which of the following statements is true?

   A. $(x + 3)$ is a non-canceling factor in the numerator. X

   B. The ratio of the constant terms of the numerator and denominator is 3. X

   C. $(x - 3)$ is a canceling factor. X

   ✓ D. $(x - 3)$ is a non-canceling factor in the numerator.

   E. $(x - 3)$ is a non-canceling factor in the denominator. X

9. Which of the following statements is true about the function $f(x) = \frac{x^2-4}{x^2+2x-3}$? $= \dfrac{(x-2)(x+2)}{(x+3)(x-1)}$

   A. $f(x)$ has two values of x at which point discontinuities exist. NO

   B. $f(x)$ is continuous for all values of x. NO

   C. $f(x)$ has one value of x at which a jump discontinuity exists. NO

   D. $f(x)$ has two values of x at which infinite discontinuities exist. True

   E. $f(x)$ has one point discontinuity and one infinite discontinuity.

10. Which of the following rational equations could be the function graphed?

A. $f(x) = \frac{2x(x-3)}{(x+2)(x-3)}$

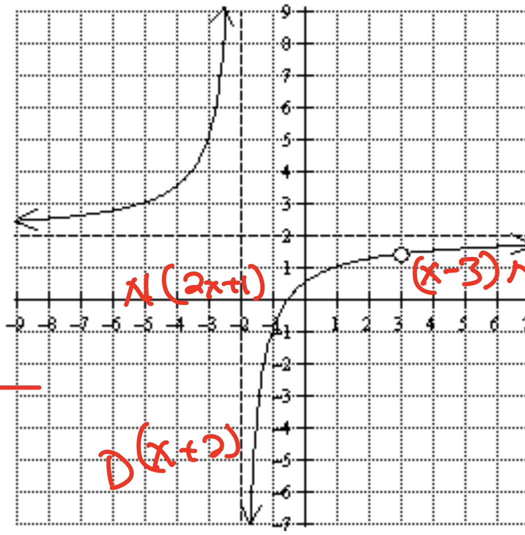
B. $f(x) = \frac{(2x+1)(x+3)}{(x+3)(x+2)}$

C. $f(x) = \frac{2x(x+2)}{(x+2)(x-3)}$

D. $f(x) = \frac{(2x+1)(x-3)}{(x-3)(x+2)}$

E. $f(x) = \frac{(2x+1)(x-3)}{(x-3)(x-2)}$

$$\frac{(2x+1)(x-3)}{(x-3)(x+2)}$$



11. Solve the rational inequality $\frac{3}{x-2} > \frac{2}{x^2-4}$.

$$\frac{(x+2)\,3}{(x+2)(x-2)} - \frac{2}{(x-2)(x+2)} > 0$$

A. $(-\infty,-2) \cup \left(-\frac{4}{3},2\right)$

B. $(-\infty,-2] \cup \left[-\frac{4}{3},2\right]$

C. $\left(-2,-\frac{4}{3}\right] \cup (2,\infty)$

D. $(-\infty,-2) \cup \left[-\frac{4}{3},2\right)$

E. $\left(-2,-\frac{4}{3}\right) \cup (2,\infty)$

$$\frac{3x+6-2}{(x-2)(x+2)} > 0$$

$$\frac{3x+4}{(x-2)(x+2)} > 0$$

- - - - und + + + + 0 - - - - - und + + + + +

-2 $\quad -\frac{4}{3}$ $\quad 2$

12. If $h(x) = \frac{(1-5x)(2x+2)}{(x+2)(2x-3)}$, then what is the equation of the horizontal asymptote, if one exists?

A. $y = -5$

B. $y = 1$

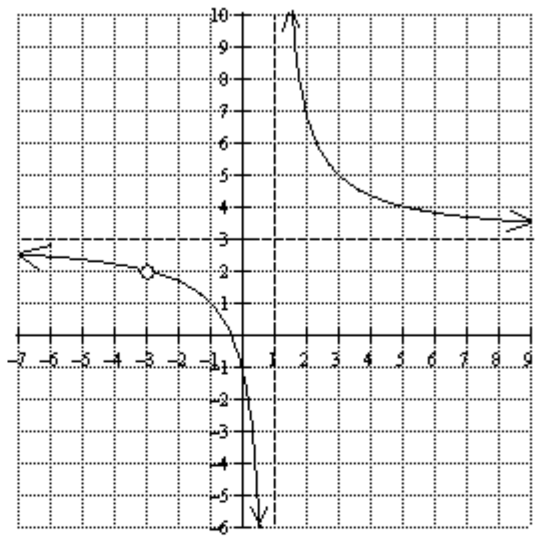
C. $y = -\frac{5}{2}$

D. $y = \frac{5}{3}$

E. No horizontal asymptote exists

$$h(x) = \frac{-10x^2 + \cdots}{2x^2 + \cdots}$$

For questions 13 – 14, refer to the graph of the rational function pictured.



VA @ x=1 ∴ (x−1) D

13. Which of the following factors is/are non-removable factors of the denominator of $f(x)$?

I. $(x - 1)$ II. $(x + 3)$ III. $(x - 3)$

A. I only
~~B. II only~~
C. I and II only
D. III only
E. II and III only

14. Which of the following statements is/are true about the function, $f(x)$? HA exists @ y=3

I. The degree of the numerator is equal to the degree of the denominator in the equation of $f(x)$. True

II. The factor ~~$(x - 3)$~~ (x+3) appears in both the numerator and the denominator of the equation of $f(x)$. False

III. If a and b are the leading coefficients of the numerator and denominator, respectively, then the value of $\frac{a}{b} = 3$. True HA @ y=3

A. I and III only B. I and II only C. III only

D. II and III only E. I, II, and III