Notes 11.3 Analyzing the Graphs of the Trig Functions

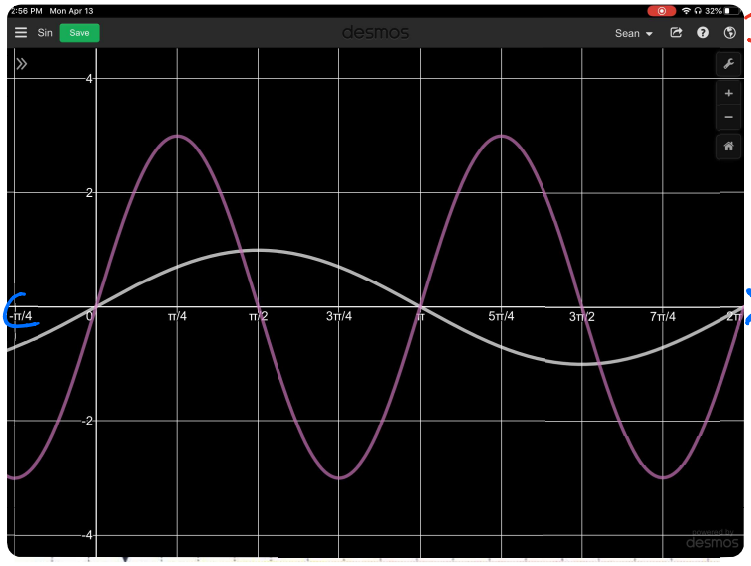
In this lesson, you will discover what happens to the graphs of the trig functions when certain parameters of the equation change. Remember the shifts, stretches, and reflection rules that we performed to the mother functions of earlier investigated functions.

The following three functions need to be graphed using the graphing calculator. Let's set the calculator to get ready to be able to do this.

1. set in radian mode.
2. hit the 2ND and WINDOW keys to take you to the TABLE SETUP.
   a. set TblStart = 0
   b. set ΔTbl = π/4.
3. hit the WINDOW key and
   a. set Xmin = 0 & Xmax = 4π,
   b. set Ymin = –4 & Ymax = 4.

Graph each function below on the indicated interval. For each function, state the range, period, amplitude, vertical stretching, horizontal stretching, vertical shifting, and/or horizontal shifting that the mother function undergoes to obtain the graph of the function.

1. Graph $f(\theta) = 3\sin(2\theta)$ on the interval [0, 2π]



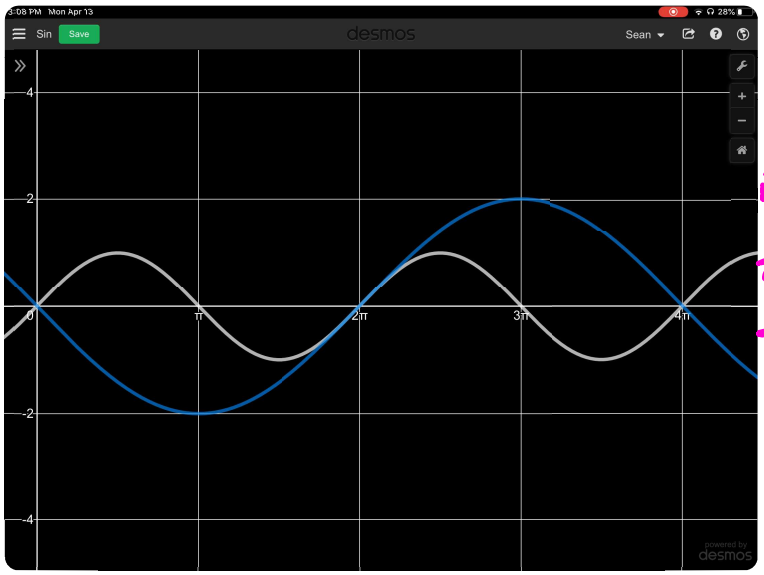
In relation to $y = \sin\theta$

1) Dilate vertically by 3
2) Dilate horizontally by $\frac{1}{2}$

MIDLINE

$R = [-3, 3]$
$P = \pi$
$A = |3| = 3$

2. Graph $g(\theta) = 2\sin\left(\frac{1}{2}\theta + \pi\right)$ on the interval [0, 4π]



$g(\theta) = 2\sin\left[\frac{1}{2}(\theta + 2\pi)\right]$

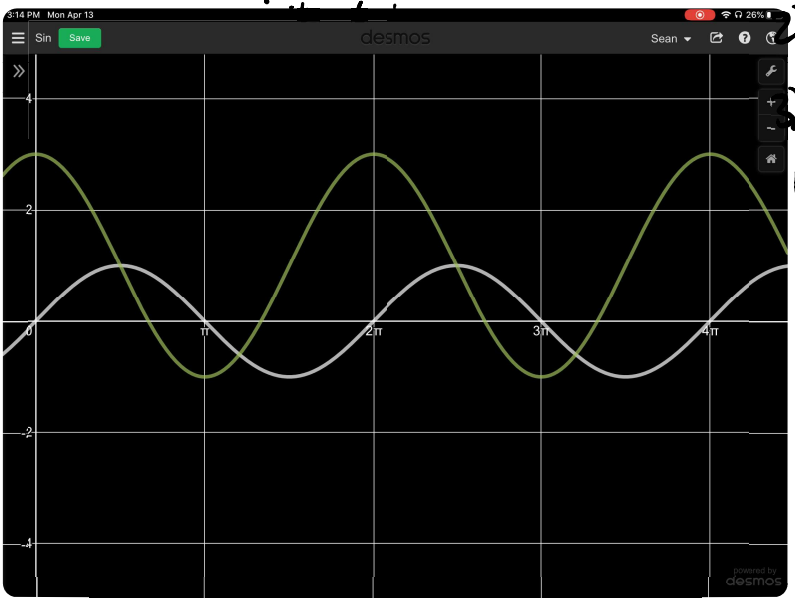
1) vertical Dilation by 2
2) Horizontal Dilation by 2
3) Horizontal translation left 2π

$P = 4\pi$
$R = [-2, 2]$
$A = |2| = 2$

3. Graph $h(\theta) = -2\sin\left(\theta - \frac{\pi}{2}\right) + 1$ on the interval $[0, 2\pi]$

Reflect vertically ↓ P.S. ↓ V.D.

1) vertical reflection
2) vertical Dilate by 2
3) vertical Displacement by 1
1) Phase Shift right $\frac{\pi}{2}$



$R = [-1, 3]$

$A = |-2| = 2$

$P = 2\pi$

Based on your observations, what graphical properties of $f(\theta) = \sin\theta$ do the values of a and b affect when the function $g(\theta) = a \cdot \sin(b\theta)$ is graphed?

"a" is going to affect the range (vertical Dilation and reflection)

"b" affects the period (Horizontal Dilation)

If possible, identify the period, amplitude, and range of each of the functions below.

a. $f(\theta) = 3\cos(4\theta)$

$P = \frac{2\pi}{4} = \frac{\pi}{2}$

$A = |3| = 3$

$R = [-3, 3]$

b. $g(\theta) = 2\cos\left(\frac{1}{4}\theta + 2\right)$

$\frac{1}{4}(\theta + 8)$

$P = 2\pi \cdot 4$

$P = 8\pi$

$A = |2| = 2$

$R = [-2, 2]$

c. $f(\theta) = -2\cos(2\theta) + 3$

$P = \pi$

$A = |-2| = 2$

$R = [1, 5]$

MIDLINE $y = 3$ (5 ↑, 1 ↓)

d. $h(\theta) = 3\sec\left(\frac{1}{2}\theta\right) - 4$

$P = 4\pi$

$R = (-\infty, -7] \cup [-1, \infty)$

$y = -4$ (−1 ↑, −7 ↓)

e. $g(\theta) = \tan\left(\frac{1}{3}\theta\right) + 2$

$P = \pi \cdot 3$

$P = 3\pi$

$R = \mathbb{R}$

f. $f(\theta) = 3\csc(2\theta - 3\pi) - 1 = 3\csc\left[2\left(\theta - \frac{3\pi}{2}\right)\right] - 1$

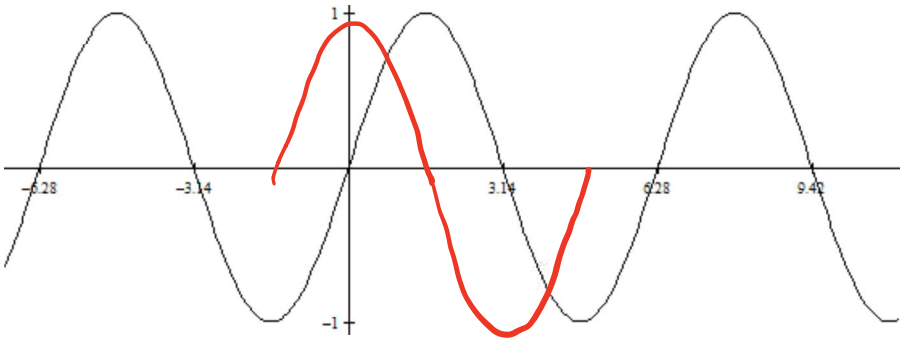
$P = 2\pi \cdot \frac{1}{2}$

$P = \pi$

$R = (-\infty, -4) \cup (2, \infty)$

$y = -1$ (? ↑, −4 ↓)

Pictured below is the graph of a trigonometric function, $f(\theta)$. Use the graph to answer the following questions.



1. Identify which basic trigonometric function is graphed above.

$f(\theta) = \sin(\theta)$

2. What is the period of the function $g(\theta) = 2 + f(\frac{1}{2}\theta)$?

$P = 4\pi$

3. What are the domain and range of the function $g(\theta) = 2 + f(\frac{1}{2}\theta)$?

$D = \mathbb{R}$ $R = [1, 3]$ $y = 3, 1, 2, \downarrow, 1$

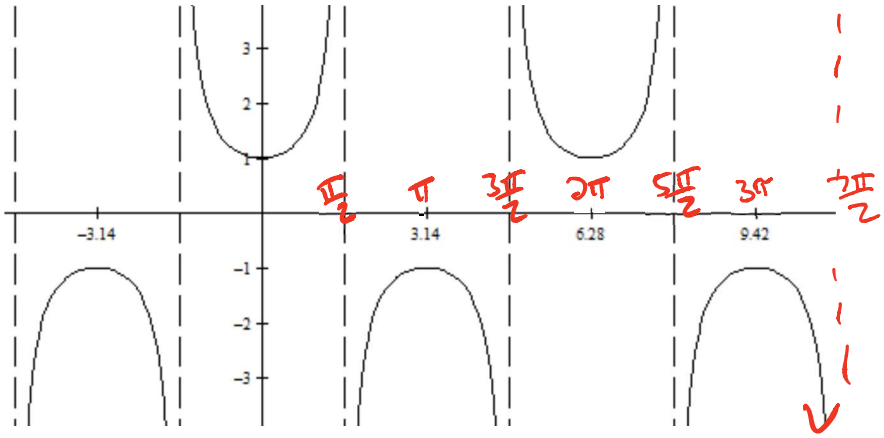
4. What is the range of the function $h(\theta) = -3 + 3f(\theta)$?

$y = 3, \uparrow, -3, \downarrow, -6$ $R = [-6, 0]$

5. Suppose the graph of $k(\theta) = f(\theta + \pi/2)$ were graphed. Which trigonometric function would result? Explain your reasoning.

$k(\theta) = \cos\theta$ because if you translate $\sin\theta$ left by $\pi/2$, it coincides with $\cos\theta$.

Pictured below is the graph of a trigonometric function, $f(\theta)$. Use the graph to answer the following questions.



6. Identify the basic function graphed above.

$f(\theta) = \sec\theta$

7. Identify the domain and range of the function graphed above.

$D = \mathbb{R}$ except $\left(\frac{\pi}{2} + \pi k\right)$ where $k \in \mathbb{Z}$

$R = (-\infty, -1] \cup [1, \infty)$

8. Identify the range of the function $g(\theta) = 2f(4\theta) + 3$.

$y = 3 \uparrow 2 \uparrow 5$

$y = 3 \downarrow 2 \downarrow 1$

$R = (-\infty, 1] \cup [5, \infty)$

9. Identify the period of the function $g(\theta) = 2f(4\theta) + 3$.

$P = 2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$

10. Describe the behavior of the graph of $f(\theta)$ as $\theta \to \frac{7\pi}{2}$ from the left. Explain your reasoning.

At $\theta = \frac{7\pi}{2}$, $f(\theta)$ has a V.A.

$\lim_{x \to \frac{7\pi}{2}^-} f(\theta) = -\infty$

As $x \to \frac{7\pi}{2}$ from the left the graph goes down forever.