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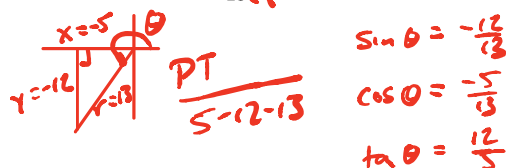
Date _____

Period _____

Homework 10.4

✓ For problems 1 and 2, an angle θ is described. Draw and label the reference triangle for each angle and then find the exact values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

1. $\cos \theta = -\frac{5}{13}$ and θ terminates in Quadrant III



$$\sin 2\theta = 2 \sin \theta \cos \theta \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \left(-\frac{12}{13}\right) \left(-\frac{5}{13}\right) = \left(-\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2$$

$$\sin 2\theta = \frac{120}{169}$$

$$= \frac{25}{169} - \frac{144}{169}$$

$$\cos 2\theta = \frac{-119}{169}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

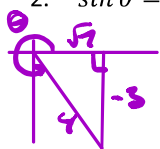
$$= \frac{2 \cdot \left(\frac{12}{5}\right)}{1 - \left(\frac{12}{5}\right)^2}$$

$$= \frac{24}{25 - 144}$$

$$= \frac{24}{-119}$$

$$\tan 2\theta = \frac{120}{-119}$$

2. $\sin \theta = -\frac{3}{4}$ and θ terminates in Quadrant IV



$$x^2 + y^2 = r^2$$

$$x^2 + (-3)^2 = (4)^2$$

$$x^2 + 9 = 16$$

$$x^2 = 7$$

$$x = \pm \sqrt{7}$$

$$\sin \theta = -\frac{3}{4}$$

$$\cos \theta = \frac{\sqrt{7}}{4}$$

$$\tan \theta = \frac{-3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(-\frac{3}{4}\right) \left(\frac{\sqrt{7}}{4}\right)$$

$$\sin 2\theta = \frac{-6\sqrt{7}}{16}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{\sqrt{7}}{4}\right)^2 - \left(-\frac{3}{4}\right)^2$$

$$= \frac{7}{16} - \frac{9}{16}$$

$$= -\frac{2}{16}$$

$$\cos 2\theta = -\frac{1}{8}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(-\frac{3}{\sqrt{7}}\right)}{1 - \left(-\frac{3}{\sqrt{7}}\right)^2} = \frac{-\frac{6}{\sqrt{7}}}{1 - \frac{9}{7}}$$

$$= \frac{-\frac{6}{\sqrt{7}}}{-\frac{2}{7}} = \frac{-6}{\sqrt{7}} \cdot \frac{7}{-2} = \frac{21}{\sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$$

$$\tan 2\theta = 3\sqrt{7}$$

Verify that the equation below is a trigonometric identity.

3. $\frac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$

$$\frac{2 \sin \theta \cos \theta}{1 - (\cos^2 \theta - \sin^2 \theta)} =$$

$$\frac{2 \sin \theta \cos \theta}{1 - \cos^2 \theta + \sin^2 \theta} =$$

$$\frac{2 \sin \theta \cos \theta}{\sin^2 \theta + \sin^2 \theta} =$$

$$\frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta} =$$

$$\frac{\cos \theta}{\sin \theta} =$$

$$\cot \theta = \cot \theta$$

4. $\cot \theta + \tan \theta = 2 \csc 2\theta$

$$\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = 2 \cdot \frac{1}{\sin 2\theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = \frac{2}{2 \sin \theta \cos \theta}$$

$$\frac{1}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} \quad \checkmark$$

$$5. \cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$

$$\cos(2\theta + 2\theta) =$$

$$\cos 2\theta \cos 2\theta - \sin 2\theta \sin 2\theta =$$

$$(\cos^2\theta - \sin^2\theta)(\cos^2\theta - \sin^2\theta) - (2\sin\theta\cos\theta)(2\sin\theta\cos\theta) =$$

$$(\cos^2\theta - (1 - \cos^2\theta))(\cos^2\theta - (1 - \cos^2\theta)) - 4\sin^2\theta\cos^2\theta =$$

$$(\cos^2\theta - 1 + \cos^2\theta)(\cos^2\theta - 1 + \cos^2\theta) - 4(1 - \cos^2\theta)\cos^2\theta =$$

$$(2\cos^2\theta - 1)(2\cos^2\theta - 1) - 4\cos^2\theta + 4\cos^4\theta =$$

$$4\cos^4\theta - 2\cos^2\theta - 2\cos^2\theta + 1 - 4\cos^2\theta + 4\cos^4\theta =$$

$$8\cos^4\theta - 8\cos^2\theta + 1 = 8\cos^4\theta - 8\cos^2\theta + 1$$

$$6. \frac{\cos(a-b)}{\cos a \sin b} = \tan a + \cot b$$

$$\frac{\cos a \cos b + \sin a \sin b}{\cos a \sin b} = \frac{\sin a \sin b}{\cos a \sin b} + \frac{\cos b \cos a}{\sin b \cos a}$$

$$\frac{\sin a \sin b + \cos a \cos b}{\sin b \cos a} = \frac{\sin a \sin b + \cos a \cos b}{\sin b \cos a}$$

$$7. \frac{\sin(a+b)}{\cos a \cos b} = \tan a + \tan b$$

$$\frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b} = \frac{\sin a \cos b}{\cos a \cos b} + \frac{\sin b \cos a}{\cos b \cos a}$$

$$\frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b} = \frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b}$$

$$8. (\sin \theta + \cos \theta)^2 = \sin 2\theta + 1$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta =$$

$$2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta =$$

$$\sin 2\theta + 1 = \sin 2\theta + 1$$

9. $\tan \theta \sin 2\theta = 2 - 2\cos^2 \theta$

$$\frac{\sin \theta}{\cos \theta} \cdot 2 \sin \theta \cos \theta = 2(1 - \cos^2 \theta)$$

$$2 \sin^2 \theta = 2 \sin^2 \theta \quad \checkmark$$

10. $\frac{\sin 2\theta}{\sin \theta} = \frac{2}{\sec \theta}$

$$\frac{2 \cancel{\sin \theta} \cos \theta}{\cancel{\sin \theta}} = 2 \cos \theta$$

$$2 \cos \theta = 2 \cos \theta \quad \checkmark$$

11. $\frac{\cos \theta}{\sin \theta \cot \theta} = \sin^2 \theta + \cos^2 \theta$

$$\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cot \theta} = 1$$

$$\cot \theta \cdot \frac{1}{\cot \theta} = 1$$

$$1 = 1$$

12. $\csc \theta \sin 2\theta - \sec \theta = \cos 2\theta \sec \theta$

$$\frac{1}{\cancel{\sin \theta}} \cdot 2 \cancel{\sin \theta} \cos \theta - \frac{1}{\cos \theta} = \frac{\cos 2\theta}{\cos \theta}$$

$$\frac{2 \cos \theta \cos \theta}{1 \cdot \cos \theta} - \frac{1}{\cos \theta} =$$

$$\frac{2 \cos^2 \theta - 1}{\cos \theta} =$$

$$\frac{\cos 2\theta}{\cos \theta} = \frac{\cos 2\theta}{\cos \theta}$$

$$13. \frac{2 \tan B}{\sin 2B} = \sec^2 B$$

$$\frac{\cancel{\cos B} \cdot 2 \cdot \frac{\sin B}{\cancel{\cos B}}}{\cancel{\cos B} \cdot 2 \sin B \cos B} =$$

$$\frac{\cancel{2} \cancel{\sin B}}{\cancel{2} \sin B \cos B} =$$

$$\frac{1}{\cos^2 B} =$$

$$\sec^2 B = \sec^2 B$$