Free Response Practice #44
Calculator Permitted

Consider the trigonometric identity $\frac{\cot\theta+1}{\cot\theta-1}=\frac{1+\tan\theta}{1-\tan\theta}$ to answer the following questions.

a. Explain why $\theta=\frac{\pi}{4}$ would not be an appropriate value to use in order to numerically validate that the given equation is an identity. Show the work that validates your reasoning.

$\frac{\cot(\frac{\pi}{4})+1}{\cot(\frac{\pi}{4})-1}=\frac{1+\tan(\frac{\pi}{4})}{1-\tan(\frac{\pi}{4})}$

$\frac{1+1}{1-1}=\frac{1+1}{1-1}$

$\frac{2}{0}=\frac{2}{0}$

und = und

When $\theta=\frac{\pi}{4}$ both sides of the equation are undefined, thus using $\theta=\frac{\pi}{4}$ would not be an appropriate value to validate the equation is an identity.

b. Based on your result from part a), what should the graph of $f(\theta)=\frac{1+\tan\theta}{1-\tan\theta}$ look like at $\theta=\frac{\pi}{4}$.

Since $f(\theta)=\frac{1+\tan\theta}{1-\tan\theta}$ is undefined at $\theta=\frac{\pi}{4}$, the graph of $f(\theta)$ is discontinuous at $\theta=\frac{\pi}{4}$. Specifically, $f(\theta)$ has a vertical asymptote at $\theta=\frac{\pi}{4}$.

Consider the trigonometric identity $\frac{\cot\theta+1}{\cot\theta-1}=\frac{1+\tan\theta}{1-\tan\theta}$ to answer the following questions.

c. Choose and use a value for θ on the interval $0<\theta<\frac{\pi}{2}$ to show that the equation is an identity. Round your answers to three decimal places.

If $\theta=\pi/3$

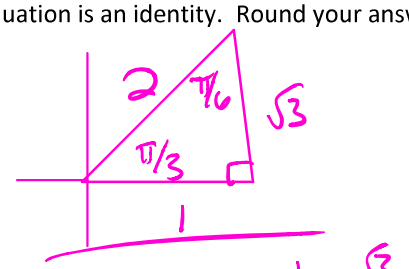
$$\frac{\cot\left(\frac{\pi}{3}\right)+1}{\cot\left(\frac{\pi}{3}\right)-1}=\frac{1+\tan\left(\frac{\pi}{3}\right)}{1-\tan\left(\frac{\pi}{3}\right)}$$

$$\frac{\frac{\sqrt{3}}{3}+\frac{3}{3}}{\frac{\sqrt{3}}{3}-\frac{3}{3}}=\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$\frac{\frac{\sqrt{3}+3}{3}}{\frac{\sqrt{3}-3}{3}}=$$

$$\frac{\sqrt{3}+3}{\sqrt{3}-3}=\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$-3.732=-3.732$$



$$\cot\left(\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$$

d. Analytically show that the equation is an identity.

$$\frac{\cot\theta+1}{\cot\theta-1}=\frac{1+\tan\theta}{1-\tan\theta}$$

$$\frac{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\sin\theta}}=\frac{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\frac{\cos\theta+\sin\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta}}=\frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}\cdot\frac{\cos\theta}{\cos\theta}$$

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

Free Response Practice #44 Grading Rubric

Free Response Part A – 2 points total

____ 1 $\theta = \frac{\pi}{4}$ would not be an appropriate value to numerically validate that the equation is an identity because both sides of the equation are undefined at $\theta = \frac{\pi}{4}$.

____ 1 Work is shown: $\frac{\cot\frac{\pi}{4}+1}{\cot\frac{\pi}{4}-1} = \frac{1+1}{1-1} = \frac{2}{0} = \text{undefined}$ OR $\frac{1+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{4}} = \frac{1+1}{1-1} = \frac{2}{0} = \text{undefined}$

Free Response Part B – 1 points total

____ 1 Since $f(\theta) = \frac{1+\tan\theta}{1-\tan\theta}$ is undefined at $f\left(\frac{\pi}{4}\right)$, then the graph of $f(\theta)$ has a vertical asymptote at $\theta = \frac{\pi}{4}$.

Free Response Part C – 3 points total

____ 1 Chooses an appropriate value, such as $\theta = \frac{\pi}{6}$ or $\frac{\pi}{3}$, which is on the interval $\left(0, \frac{\pi}{2}\right)$ and evaluates both sides of the equation at that value.

____ 1 Work for $\frac{\cot\theta+1}{\cot\theta-1}$: $\frac{\cot\frac{\pi}{6}+1}{\cot\frac{\pi}{6}-1} = 3.732$ OR $\frac{\cot\frac{\pi}{3}+1}{\cot\frac{\pi}{3}-1} = -3.732$

____ 1 Work for $\frac{1+\tan\theta}{1-\tan\theta}$: $\frac{1+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{6}} = 3.732$ OR $\frac{1+\tan\frac{\pi}{3}}{1-\tan\frac{\pi}{3}} = -3.732$

Free Response Part D – 3 points total

____ 1 Correctly rewrites both sides in terms of sine and cosine

____ 1 Correctly simplifies $\frac{\cot\theta+1}{\cot\theta-1}$

____ 1 Correctly simplifies $\frac{1+\tan\theta}{1-\tan\theta}$

$$\frac{\cot\theta+1}{\cot\theta-1} = \frac{1+\tan\theta}{1-\tan\theta}$$

$$\frac{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\sin\theta}} = \frac{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\cos\theta+\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta-\sin\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$