Free Response Practice #43
Calculator Permitted

Suppose that θ is an angle in standard position such that $\sin\theta = \frac{12}{13}$ and $\cos\theta < 0$. Additionally, λ is a different angle in standard position such that $\sec\lambda = -\frac{5}{3}$ and $\sin\lambda < 0$.

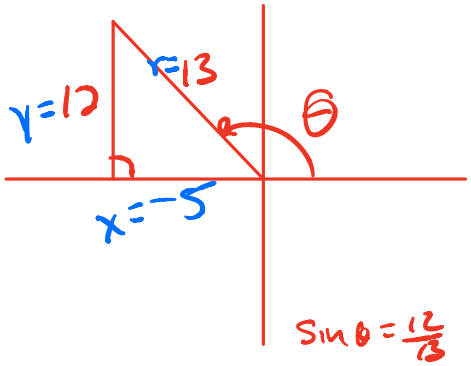
a. In which quadrant does the terminal side of θ lie? Completely explain your reasoning.

The terminal side of θ lies in quadrant II.
Since $\sin\theta > 0$ and $\cos\theta < 0$, this implies y is + and x is −. Quadrant II has (−, +)

b. In which quadrant does the terminal side of λ lie? Completely explain your reasoning.

The terminal side of γ lies in quadrant III.
Since $\sec\gamma = -$, then $x = -$
Since $\sin\gamma = -$, then $y = -$.
In QUAD III, both x and y are negative.

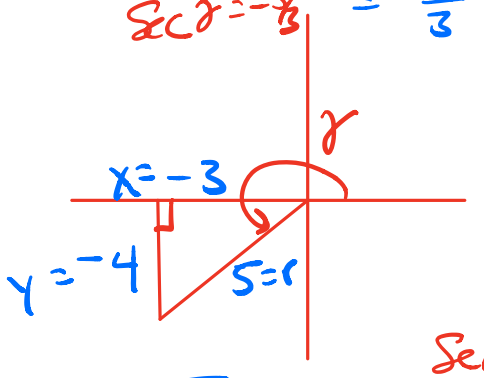
c. Draw and completely label the reference triangles for both θ and λ.



PT 5-12-13

$\sin\theta = \frac{12}{13}$
$\cos\theta = \frac{-5}{13}$

$\sec\gamma = -\frac{5}{3} = \frac{5}{-3} = -\frac{5}{3}$



PT 3-4-5

$\sec\gamma = -\frac{5}{3}$
$\cos\gamma = -\frac{3}{5}$
$\sin\gamma = -\frac{4}{5}$

d. Which of the following, $\sin(\theta + \lambda)$ or $\cos(\theta - \lambda)$, has the greater value? Show your work.

$\sin\theta = \frac{12}{13}$ $\quad \cos\gamma = \frac{-3}{5}$

$\cos\theta = \frac{-5}{13}$ $\quad \sin\gamma = \frac{-4}{5}$

$\sin(\theta+\gamma) = \sin\theta\cos\gamma + \sin\gamma\cos\theta$

$= \frac{12}{13}\left(\frac{-3}{5}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-36}{65} + \frac{20}{65}$

$\sin(\theta+\gamma) = \frac{-16}{65}$

$\cos(\theta-\gamma) = \cos\theta\cos\gamma + \sin\theta\sin\gamma$

$= \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right)$

$= \frac{15}{65} + \frac{-48}{65}$

$\cos(\theta-\gamma) = \frac{-33}{65}$

$\cos(\theta-\gamma) < \sin(\theta-\gamma)$

Free Response Practice #43 Grading Rubric

Free Response Part A – 2 points total

____ 1 Since $\sin\theta = \frac{y}{r} > 0$, then $y > 0$ because r is always positive. Additionally, $\cos\theta < 0$, which means that $x < 0$.

____ 1 Since $x < 0$ and $y > 0$, then the angle θ terminates in Quadrant II.

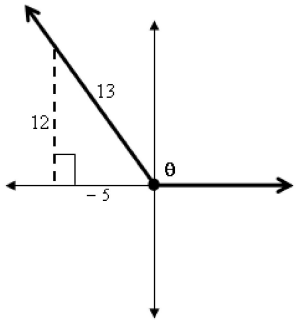
Free Response Part B – 2 points total

____ 1 Since $\sec\lambda = \frac{r}{x} < 0$, then $x < 0$ because r is always positive. Additionally, $\sin\lambda < 0$, which means that $y < 0$.

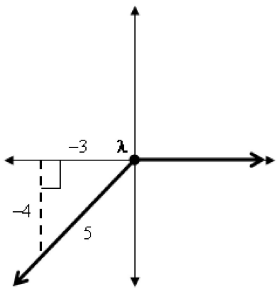
____ 1 Since $x < 0$ and $y < 0$, then the angle λ terminates in Quadrant III.

Free Response Part C – 2 points total

____ 1 Correct reference triangle for θ



____ 1 Correct reference triangle for λ



Free Response Part D – 3 points total

____ 1 Finds correct value for $\cos(\theta - \lambda) = \cos\theta\cos\lambda + \sin\theta\sin\lambda = -\frac{33}{65} = -0.507$

____ 1 Finds correct value for $\sin(\theta + \lambda) = \sin\theta\cos\lambda + \cos\theta\sin\lambda = -\frac{16}{65} = -0.246$

____ 1 $\cos(\theta - \lambda) < \sin(\theta + \lambda)$