Free Response Practice #41
Calculator NOT Permitted

Answer the following questions using trigonometric identities. Be sure to show every step of your work.

a. Show that the expression $sec\,\theta\, csc\,\theta - tan\,\theta$ can be simplified to $\boxed{cot\,\theta}$.

$$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$$

SIDE WORK

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$= \frac{\cos^{\cancel{2}}\theta}{\sin\theta\cancel{\cos\theta}}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

b. Show that the expression $\frac{sec\,\theta + csc\,\theta}{1 + tan\,\theta}$ can be simplified to $csc\,\theta$.

$$= \frac{\frac{\sin\theta}{\sin\theta}\cdot\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta + \cos\theta}{\cancel{\sin\theta\cdot\cos\theta}}}{\frac{\cos\theta + \sin\theta}{\cancel{\cos\theta}}}$$

$$\frac{\cancel{\sin\theta}\,\cancel{\cos\theta}}{\sin\theta\,\cancel{\cos\theta}}$$

$$= \frac{\cancel{(\sin\theta + \cos\theta)}\cdot 1}{\cancel{(\cos\theta + \sin\theta)}\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

c. Verify that the equation $\frac{(\sec\theta+1)}{(\sec\theta+1)}\cdot\frac{\sec\theta-1}{\tan\theta}+\frac{\tan\theta}{\sec\theta+1}\cdot\frac{\tan\theta}{\tan\theta}=\frac{2\sin\theta}{1+\cos\theta}$ is an identity.

$$\frac{(\sec\theta+1)(\sec\theta-1)+\tan\theta\tan\theta}{(\sec\theta+1)\tan\theta}=$$

$$\frac{(\sec^2\theta)-1+\tan^2\theta}{(\sec\theta+1)\tan\theta}=$$

$$\frac{1+\tan^2\theta\ -1+\tan^2\theta}{(\sec\theta+1)\tan\theta}=$$

$$\frac{2\tan^{\cancel{2}}\theta}{(\sec\theta+1)\cancel{\tan\theta}}=$$

$$\frac{2\tan\theta}{\sec\theta+1}=$$

$$\frac{2\cdot\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}=$$

$$\frac{\cos\theta}{\cos\theta}\cdot\frac{2\cdot\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1+\cos\theta}{\cancel{\cos\theta}}}=$$

$$\frac{2\sin\theta}{1+\cos\theta}=\frac{2\sin\theta}{1+\cos\theta}$$

Free Response Practice #41 Grading Rubric

Free Response Part A – 3 points total

____ 1 Rewrites the entire expression correctly in terms of sin θ and cos θ

____ 1 Shows all steps of the correct, algebraic simplification

____ 1 Logically, through use of trig identities, arrives at the answer cot θ

$$\sec\theta\csc\theta - \tan\theta$$
$$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$
$$\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{\sin\theta}\right)$$
$$\frac{1}{\cos\theta\sin\theta} - \frac{\sin^2\theta}{\cos\theta\sin\theta}$$
$$\frac{1-\sin^2\theta}{\cos\theta\sin\theta}$$
$$\frac{\cos^2\theta}{\cos\theta\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Free Response Part B – 3 points total

____ 1 Rewrites the entire expression correctly in terms of sin θ and cos θ

____ 1 Shows all steps of the correct, algebraic simplification

____ 1 Logically, through use of trig identities, arrives at the answer csc θ

$$\frac{\frac{1}{\cos\theta}+\frac{1}{\sin\theta}}{1+\frac{\sin\theta}{\cos\theta}} = \frac{\frac{1}{\cos\theta}\left(\frac{\sin\theta}{xin\theta}\right)+\frac{1}{\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)}{\left(\frac{\cos\theta}{\cos\theta}\right)+\frac{\sin\theta}{\cos\theta}}$$
$$= \frac{\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$
$$= \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\cdot\frac{\cos\theta}{\cos\theta+\sin\theta}$$
$$= \frac{\cos}{\cos\theta\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$$

Free Response Part C – 3 points total

____ 1 Correctly substitutes trig identities at all appropriate places

____ 1 Shows all steps of the correct, algebraic simplification

____ 1 Logically, through use of trig identities, shows that both sides of the equation simplify to the same trig expression sides of the equation simplify to the same trig expression

$$\frac{\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x} = 2\tan x$$
$$\frac{\cos x}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right) - \frac{\cos x}{1+\sin x}\left(\frac{1-\sin x}{1-\sin x}\right) = 2\tan x$$
$$\frac{\cos x+\cos x\sin x}{(1-\sin x)(1+\sin x)} - \frac{\cos x-\cos x\sin x}{(1-\sin x)(1+\sin x)} = \frac{2\sin x}{\cos x}$$
$$\frac{\cos x+\cos x\sin x-\cos x+\cos x\sin x}{(1-\sin x)(1+\sin x)} = \frac{2\sin x}{\cos x}$$
$$\frac{2\cos x\sin x}{1-\sin^2 x} = \frac{2\sin x}{\cos x}$$
$$\frac{2\cos x\sin x}{\cos^2 x} = \frac{2\sin x}{\cos x}$$
$$\frac{2\sin x}{\cos x} = \frac{2\sin x}{\cos x}$$