FREE RESPONSE – Calculator Permitted

Suppose that $\cot\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$ while $\cos\alpha = -\frac{3}{5}$ and $\pi < \alpha < \frac{3\pi}{2}$.

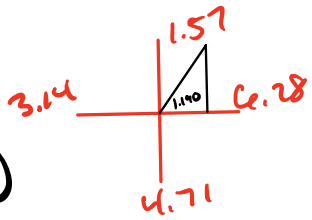
a. To the nearest thousandth of a radian, find the values of θ and α. Show your work

$\cot\theta = \frac{2}{5}$

$\tan\theta = \frac{5}{2}$

$\theta = \tan^{-1}(5/2)$

$\theta = 1.190$ (QI) +1



$\cos\alpha = -\frac{3}{5}$

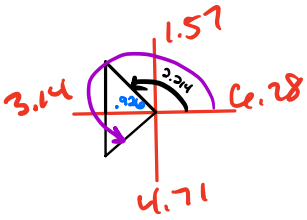
$\alpha = \cos^{-1}(-3/5)$

$\alpha = 2.214$ +1

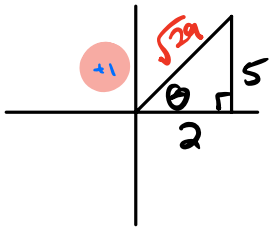
$\alpha = \pi + .926$

$\alpha = 4.068$ (QIII)

+1



b. Draw and label the reference triangles for both θ and α.



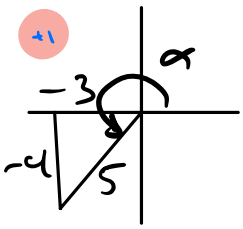
$x^2 + y^2 = r^2$

$(2)^2 + (5)^2 = r^2$

$4 + 25 = r^2$

$29 = r^2$

$\pm\sqrt{29} = r$



PT
3-4-5

c. Find the value of $\tan(\theta + \alpha)$. Show your work using an identity and leave your answer in exact form.

$\tan(\theta+\alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$ +1

$= \frac{6\cdot\frac{5}{2} + \frac{4}{3}6}{6\cdot 1 - (5/2)(4/3)6}$

$= \frac{15 + 8}{6 - 20}$

$\tan(\theta+\alpha) = \frac{23}{-14}$ +1

d. Find the value of $\sin(2\theta)$. Show your work using an identity and leave your answer in exact form.

$\sin(2\theta) = 2\sin\theta\cos\theta$ +1

$= 2\left(\frac{5}{\sqrt{29}}\right)\left(\frac{2}{\sqrt{29}}\right)$

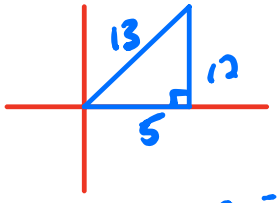
$= \frac{20}{(\sqrt{29})^2}$

$\sin(2\theta) = \frac{20}{29}$ +1

MULTIPLE CHOICE – Calculator Permitted

1. If $\cot\theta = \frac{5}{12}$, find $\csc\theta$ if $0° < \theta < 90°$.

A. $\frac{12}{13}$ B. $\frac{5}{12}$ C. $-\frac{5}{12}$ D. $\frac{13}{12}$ E. -1



PT 5-12-13

$\csc\theta = \frac{r}{y} = \frac{13}{12}$

2. Which of the following equation(s) is/are true for all values of θ?

I. $\tan\theta = \sin\theta\sec\theta$ II. $\tan\theta\sin\theta = \cos\theta$ III. $\sin\theta\,(\cot\theta + \tan\theta) = \sec\theta$

A. I only B. II only C. I and III only D. III only E. II and III only

$\tan\theta = \sin\theta \cdot \frac{1}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}\sin\theta = \cos\theta$

$\sin^2\theta = \cos^2\theta$

$\sin\theta\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \sec\theta$

$\cos\theta + \frac{\sin^2\theta}{\cos\theta} = \sec\theta$

$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$\sec\theta = \sec\theta$

3. Simplify $\frac{\csc x \cos^2 x}{1+\csc x}$.

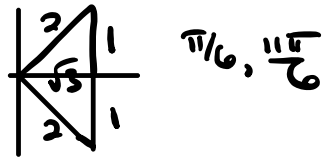
A. $\csc x + 1$ B. $1 - \sin x$ C. $\sin x - 1$ D. $1 + \sin x$ E. $\sin^2 x - 1$

$= \frac{\frac{\cos^2 x}{\sin x}}{1 + \frac{1}{\sin x}} \cdot \frac{\sin x}{\sin x} = \frac{\cos^2 x}{\sin x + 1} = \frac{1 - \sin^2 x}{\sin x + 1} = \frac{(1-\sin x)(1+\sin x)}{\sin x + 1}$

4. Find all of the solutions on the interval $0 \le x < 2\pi$ for $2\cos x - \sqrt{3} = 0$.

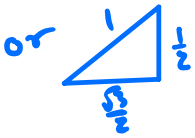
A. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$ B. $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$ C. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ D. $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ E. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

$\cos x = \frac{\sqrt{3}}{2} = r$



$\pi/6, \frac{11\pi}{6}$

or



MULTIPLE CHOICE – Calculator Permitted

5. Simplify the expression $tan\left(\frac{\pi}{4} - \theta\right)$.

A. $\frac{\sqrt{2}+2\tan\theta}{2-\sqrt{2}\tan\theta}$ B. $\frac{1-\tan\theta}{1+\tan\theta}$ C. $\frac{1+\tan\theta}{1-\tan\theta}$ D. 1 E. –1

$= \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} = \frac{1 - \tan\theta}{1 + 1\cdot\tan\theta}$

6. For $0 < \theta \leq 2\pi$, which of the following value(s) is/are solutions of the equation $6\sin^2\theta - \sin\theta = 2$?

I. 2.412 II. $\frac{7\pi}{6}$ III. $\frac{11\pi}{6}$

A. II only B. II and III only C. I and II only D. III only E. I, II and III

$6\sin^2\theta - \sin\theta - 2 = 0$

$6\sin^2\theta - 4\sin\theta + 3\sin\theta - 2 = 0$

$2\sin\theta(3\sin\theta - 2) + 1(3\sin\theta - 2) = 0$

$(3\sin\theta - 2)(2\sin\theta + 1) = 0$

$3\sin\theta - 2 = 0$: $3\sin\theta = 2$, $\sin\theta = \frac{2}{3}$

$2\sin\theta + 1 = 0$: $2\sin\theta = -1$, $\sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{2}{3}\right)$

$\theta = .730$

1.57, $\pi - .730 = 2.412$, 3.14, .730, 6.28, 4.71

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{6}$, $\frac{5\pi}{6}$, $-\frac{1}{2}$, $-\frac{1}{2}$, $\frac{11\pi}{6}$

7. The expression $\sin\theta\cos\alpha - \cos\theta\sin\alpha$ is equivalent to which of the following expressions?

A. $\sin(\theta + \alpha)$ B. $\cos(\theta + \alpha)$ C. $\sin(\theta - \alpha)$ D. $\sin(\theta + \alpha)$ E. $\sin(2\theta\alpha)$

$\sin(\theta - \alpha)$

MULTIPLE CHOICE – Calculator NOT Permitted

8. Find all of the solutions on the interval $0 < x \le 2\pi$ for $2\cos x + \sqrt{3} = 0$.

A. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

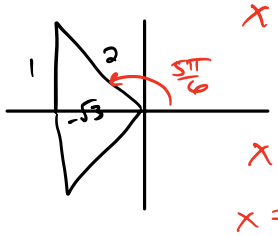
B. $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

C. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

D. $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

E. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

$\cos x = -\frac{\sqrt{3}}{2}$



$x' = \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

$x = \frac{7\pi}{6}$

9. Simplify the expression $\dfrac{\sin\left(\frac{\pi}{2}+x\right)-\sin\frac{\pi}{2}}{x}$.

A. $\frac{\sin x}{x}$

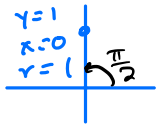
B. $\frac{\sin x - 1}{x}$

C. 1

D. $\frac{\cos x - 1}{x}$

E. $\frac{\cos x + 1}{x}$

$\dfrac{\sin\left(\frac{\pi}{2}\right)\cos x + \sin x \cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)}{x}$



$\sin\left(\frac{\pi}{2}\right) = 1$

$\cos\left(\frac{\pi}{2}\right) = 0$

$= \dfrac{1\cdot\cos x + \sin x \cdot 0 - 1}{x}$

$= \dfrac{\cos x - 1}{x}$

10. Given that $\cos\left(\frac{\pi}{2} - x\right) = \frac{2}{7}$, what is the value of $\sin^2 x$?

A. $\frac{45}{49}$

B. $\frac{49}{4}$

C. $\frac{45}{4}$

D. $\frac{4}{49}$

E. $\frac{2}{7}$

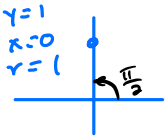
$\cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x = \frac{2}{7}$

$0\cdot\cos x + 1\cdot\sin x = \frac{2}{7}$

$\sin x = \frac{2}{7}$

$(\sin x)^2 = \left(\frac{2}{7}\right)^2$

$\sin^2 x = \frac{4}{49}$



$\sin\left(\frac{\pi}{2}\right) = 1$

$\cos\left(\frac{\pi}{2}\right) = 0$

11. The expression $\dfrac{\csc\theta}{\tan\theta+\cot\theta}$ can be simplified to which of the following expressions?

A. $\cos\theta+\tan\theta$

B. $\sin^2\theta+\cos\theta$

C. $\csc^2\theta\sec\theta$

D. $\cos\theta$

E. $\sin\theta$

$$= \frac{\frac{1}{\sin\theta}\sin\theta\cos\theta}{\frac{\sin\theta}{\cos\theta}\sin\theta\cos\theta+\frac{\cos\theta}{\sin\theta}\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin^2\theta+\cos^2\theta}$$

$$= \frac{\cos\theta}{1}$$

$$= \cos\theta$$

12. If $\tan\theta>0$ and $\sec\theta<0$, determine the quadrant in which θ lies.

A. I B. II C. III D. IV E. II or IV

$x<0$

QUAD I or III

$x>0$ $x<0$

$y>0$ $y<0$

left of x-axis

13. Determine the exact value of the expression: $\sin\frac{5\pi}{7}\cos\frac{2\pi}{7}+\cos\frac{5\pi}{7}\sin\frac{2\pi}{7}$.

($\frac{5\pi}{7}$ is not special)

$$= \sin\left(\frac{5\pi}{7}+\frac{2\pi}{7}\right)$$

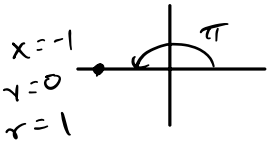
$$= \sin\left(\frac{7\pi}{7}\right)$$

$$= \sin(\pi)$$

$$= 0$$

A. 1 B. $\frac{\sqrt{3}}{2}$ C. –1

D. 0 E. $\frac{1}{2}$



14. Determine which of the following equations represents a trigonometric identity.

I. $\csc\theta\cos\theta = \cot\theta$ ✓

$\frac{1}{\sin\theta}\cos\theta = \cot\theta$

$\frac{\cos\theta}{\sin\theta} =$

$\cot\theta = \cot\theta$

II. $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$ ✓

$\cos\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) =$

$\sin\theta + \frac{\cos^2\theta}{\sin\theta} =$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta} =$

$\frac{1}{\sin\theta} =$

$\csc\theta = \csc\theta$

III. $\cos\theta\cot\theta = \csc\theta - \cos\theta$ ✗

$\cos\theta \cdot \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta\sin\theta}{\sin\theta}$

$\frac{\cos^2\theta}{\sin\theta} = \frac{1 - \sin\theta\cos\theta}{\sin\theta}$

$\frac{1-\sin^2\theta}{\sin\theta} = \frac{1 - \sin\theta\cos\theta}{\sin\theta}$