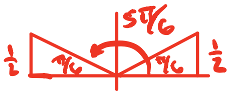
Notes 10.5 Solving Trigonometric Equations

Solve the equation, finding two values of θ such that $0 < \theta \le 2\pi$. Round your answers to the nearest thousandth of a radian if needed.

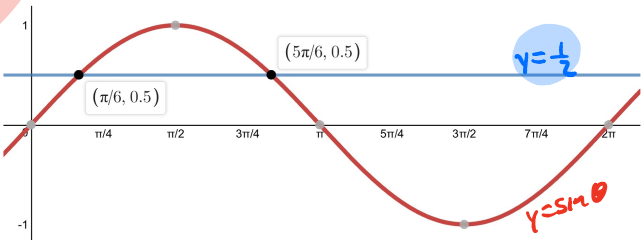
$$\sin^{-1}\left(\sin\theta\right) = \sin^{-1}\left(\frac{1}{2}\right)$$



$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin(\theta) = \frac{1}{2}$

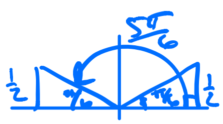


In all of the equations that we have solved thus far, the argument of the trig function in the equation is $1 \cdot \theta$. Thus, we only look for values on 1 period on the interval $0 < \theta \le 2\pi$.

When we have the equation like $\sin 2\theta = -0.792$, then the graph will make 2 periods on the interval $0 < \theta \le 2\pi$.

Solve the equations below, finding exact solutions, when possible, on the interval $0 < \theta \le 2\pi$. Round your answers to the nearest thousandth of a radian, if necessary.

1. $\sin 2\theta = \frac{1}{2}$ QUAD I, II

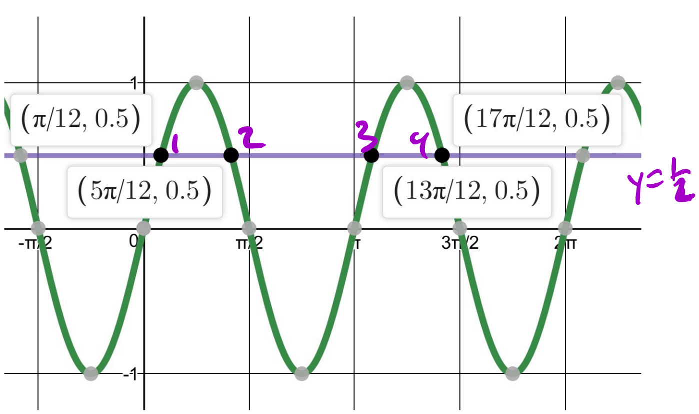


$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$

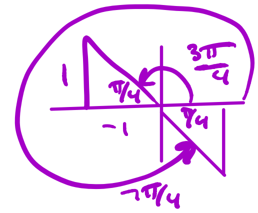
$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $+\frac{12\pi}{12}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$\frac{12\pi}{12}$



2. $\tan 2\theta = -1$ QUAD II, IV

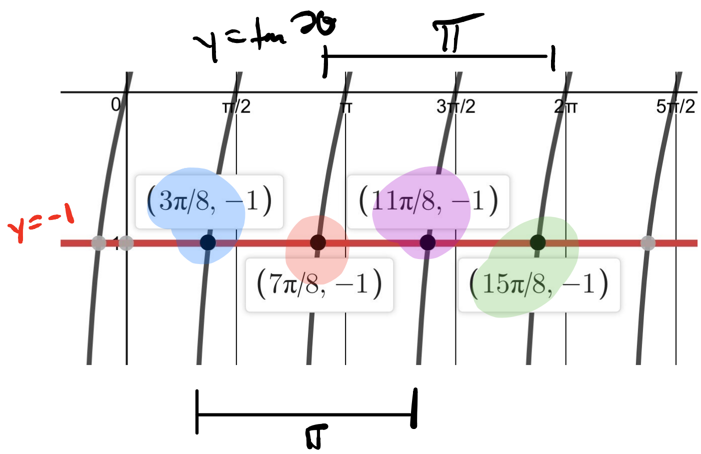


$2\theta = \tan^{-1}(-1)$

$2\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ $+\frac{8\pi}{8}$

$\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

$\frac{8\pi}{8}$

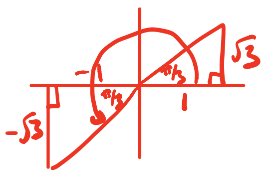


Up until now, we have only solved equations that contain a single trigonometric ratio. Often, however, we encounter equations that contain more than one. For example, the equation has both sine and cosine in it. When we encounter this, our goal is to see if we can rewrite the equation in terms of only one trigonometric ratio.

3. $\sin\theta = \sqrt{3}\cos\theta$ one trig RATIO = #

$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$ ✓

$\tan\theta = \sqrt{3} = \frac{\sqrt{3}}{1} = \frac{y}{x}$



$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \pi/3, \frac{4\pi}{3}$

4. $4\sin\theta = \csc\theta$ $\cdot \sin\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

Before we continue with this lesson, let's connect what we know about factoring polynomials in algebra to factoring trigonometric expressions. Completely factor each of the following trigonometric expressions.

5. $4\sin^2\theta - 1 = (2\sin\theta - 1)(2\sin\theta + 1)$

6. $\cos\theta\sin^2\theta - 4\cos^3\theta = \cos\theta(\sin^2\theta - 4\cos^2\theta)$

$= \cos\theta(\sin\theta - 2\cos\theta)(\sin\theta + 2\cos\theta)$

7. $\sin^2\theta - \cos^2\theta = (\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

8. $\sin^2\theta - 2\sin\theta - 15 = (\sin\theta - 5)(\sin\theta + 3)$

$M = -15$

$A = -2$

$N = -5, 3$

9. $4\tan^2\theta + \tan\theta - 3 = 4\tan^2\theta + 4\tan\theta - 3\tan\theta - 3$

$M = 4\tan^2\theta(-3) = -12\tan^2\theta$

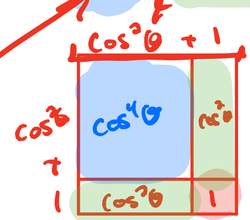
$A = \tan\theta$

$N = 4\tan\theta, -3\tan\theta$

$= 4\tan\theta(\tan\theta + 1) - 3(\tan\theta + 1)$

$= (\tan\theta + 1)(4\tan\theta - 3)$

10. $\cos^4\theta + 2\cos^2\theta + 1 = (\cos^2\theta + 1)^2$



□ tri?

1) 1st term □?

2) Last term □?

3) $2\sqrt{1^{st}}\sqrt{Last}$ = middle?

On the next series of examples, we will employ this idea of factoring to solve the trigonometric equations.
Solve the equations below, finding solutions on the interval $0 < \theta \leq 2\pi$. Give exact values of θ, when possible.

11. $2\,sin^2\,\theta = sin\,\theta + 1$

$2\sin^2\theta - \sin\theta - 1 = 0$

$2\sin^2\theta - 2\sin\theta + \sin\theta - 1 = 0$

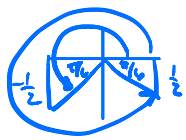
$2\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(\sin\theta - 1)(2\sin\theta + 1) = 0$

M: $-2\sin^2\theta$
A: $-\sin\theta$
N: $-2\sin\theta,\ \sin\theta$

$\sin\theta - 1 = 0$
$\sin\theta = 1$
$\theta = \sin^{-1}(1)$
$\theta = \pi/2$

$2\sin\theta + 1 = 0$
$2\sin\theta = -1$
$\sin\theta = -\frac{1}{2}$
$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$
$\theta = 7\pi/6,\ 11\pi/6$



12. $sin\,2\theta = sin\,\theta$

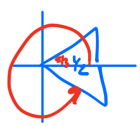
$2\sin\theta\cos\theta = \sin\theta$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$
$\theta = \sin^{-1}(0)$
$\theta = \pi, 2\pi$

$2\cos\theta - 1 = 0$
$2\cos\theta = 1$
$\cos\theta = \frac{1}{2}$
$\theta = \cos^{-1}\left(\frac{1}{2}\right)$
$\theta = \pi/3,\ 5\pi/3$



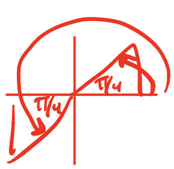
13. $tan^2\,\theta = 2 - tan\,\theta$

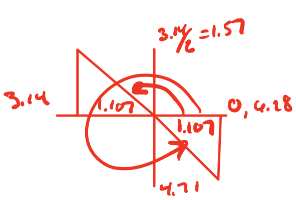
$\tan^2\theta + \tan\theta - 2 = 0$

$(\tan\theta + 2)(\tan\theta - 1) = 0$

$\tan\theta + 2 = 0$
$\tan\theta = -2$
$\theta = \tan^{-1}(-2)$
CALC
$\theta = -1.107$

$\tan\theta - 1 = 0$
$\tan\theta = 1$
$\theta = \tan^{-1}(1)$
$\theta = \pi/4,\ 5\pi/4$





$\theta = \pi - 1.107 = 2.304$

$\theta = 2\pi - 1.107 = 5.176$

Solve the equation below, utilizing sum/difference and/or double angle identities.

14. $4\,sin\,\theta\,cos\,\theta = -\sqrt{2}$

$2 \cdot 2\sin\theta\cos\theta = -\sqrt{2}$

$2\sin\theta\cos\theta = -\frac{\sqrt{2}}{2}$

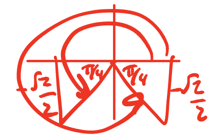
$\sin(2\theta) = -\frac{\sqrt{2}}{2}$

$2\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$2\theta = \frac{5\pi}{4},\ \frac{7\pi}{4}$

$\theta = \frac{5\pi}{8},\ \frac{7\pi}{8},\ \frac{13\pi}{8},\ \frac{15\pi}{8}$

$+\pi$ $+\pi$



15. $2\,cos\,x\,cos\,x - 2\,sin\,x\,sin\,x = \sqrt{3}$

$2(\cos^2 x - \sin^2 x) = \sqrt{3}$

$2 \cdot \cos(2x) = \sqrt{3}$

$\cos(2x) = \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2x = \frac{\pi}{6},\ \frac{11\pi}{6}$

$x = \frac{\pi}{12},\ \frac{11\pi}{12},\ \frac{13\pi}{12},\ \frac{23\pi}{12}$

$+\pi$ $+\pi$