$x = -$
$y = -$
$r = +$
III

Notes 10.4 Double Angle Identities

In this lesson, we will derive identities for $sin\ 2\theta$, $cos\ 2\theta$, and $tan\ 2\theta$ by using the sum identities that we learned in a previous lesson.

Double Angle Identity for Sine

$$\sin(2\theta) = \sin(\theta + \theta)$$
$$= \sin\theta\cos\theta + \sin\theta\cos\theta$$
$$\boxed{\sin(2\theta) = 2\sin\theta\cos\theta}$$

Double Angle Identities for Cosine

$$\cos(2\theta) = \cos(\theta + \theta)$$
$$= \cos\theta \cdot \cos\theta - \sin\theta\sin\theta$$
$$\boxed{\cos(2\theta) = \cos^2\theta - \sin^2\theta}$$

$$\cos(2\theta) = 1 - \sin^2\theta - \sin^2\theta$$
$$\boxed{\cos(2\theta) = 1 - 2\sin^2\theta}$$

$$\cos(2\theta) = \cos^2\theta - (1 - \cos^2\theta)$$
$$= \cos^2\theta - 1 + \cos^2\theta$$
$$\boxed{\cos(2\theta) = 2\cos^2\theta - 1}$$

Double Angle Identity for Tangent

$$\tan(2\theta) = \tan(\theta + \theta)$$
$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta \cdot \tan\theta}$$
$$\boxed{\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

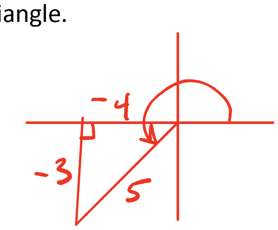
1. Suppose $sin\ \theta = -\frac{3}{5}$ and $tan\ \theta > 0$. Answer the following questions about θ.
   a. In which quadrant does θ terminate? Explain your reasoning.

- $\sin\theta = \frac{y}{r} = \frac{-3}{5}$, so $\sin\theta$ must be in QUAD III, IV.
- Since $\tan\theta > 0$, then the signs of y and x are same, which means terminal side is I, III

Thus θ terminates in Quadrant III.

   b. Draw the angle θ and label the reference triangle.

$\sin\theta = \frac{-3}{5}$



   c. Order $sin\ 2\theta$, $cos\ 2\theta$, and $tan\ 2\theta$ from least to greatest.

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$$
$$\sin(2\theta) = \frac{24}{25}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25}$$
$$\cos(2\theta) = \frac{7}{25}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$
$$= \frac{\cancel{2}\left(\frac{3}{\cancel{4}_2}\right)}{1 - \left(\frac{3}{4}\right)^2}$$
$$= \frac{\frac{3}{2}}{\frac{16}{16} - \frac{9}{16}}$$
$$= \frac{\frac{3}{2}}{\frac{7}{16}}$$
$$= \frac{3}{\cancel{2}_1} \cdot \frac{\cancel{16}^8}{7}$$
$$= \frac{24}{7}$$

$$\cos(2\theta) < \sin(2\theta) < \tan(2\theta)$$

Verify that each equation is an identity.

2. $\cot\theta - \tan\theta = 2\cot 2\theta$

$$\frac{\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta} = 2\cdot\frac{\cos 2\theta}{\sin 2\theta}$$

$$\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta} = \cancel{2}\cdot\frac{\cos 2\theta}{\cancel{2}\sin\theta\cos\theta}$$

$$\frac{\cos 2\theta}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\sin\theta\cos\theta}$$

3. $\frac{\sin 2x}{1-\cos 2x} = \cot x$

$$\frac{2\sin x\cos x}{1-(\cos^2 x-\sin^2 x)} =$$

$$\frac{2\sin x\cos x}{1-\cos^2 x+\sin^2 x} =$$

$$\frac{2\sin x\cos x}{\sin^2 x+\sin^2 x} =$$

$$\frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}\sin^{\cancel{2}} x} =$$

$$\frac{\cos x}{\sin x} =$$

$$\cot x = \cot x$$

4. $\sin 4x = 4(\sin x\cos^3 x - \sin^3 x\cos x)$

$$\sin(2x+2x) = 4\sin x\cos x(\cos^2 x - \sin^2 x)$$

$$2\sin 2x\cos 2x = 4\sin x\cos x\cdot\cos 2x$$

$$2\cdot 2\sin x\cos x\cos 2x =$$

$$4\sin x\cos x\cos 2x = 4\sin x\cos x\cdot\cos 2x$$