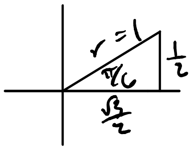
Notes 10.2 Verifying Trigonometric Identities

As we stated last time, an identity is simply an equation that states that two trigonometric expressions are equal to each other for all values of θ.

~~Consider for a moment~~ the equation $sec\,\theta - cos\,\theta = sin\,\theta\,tan\,\theta$. Show that both sides of this equation equal the same value if $\theta = \frac{\pi}{6}$.



$$\sec\theta - \cos\theta = \sin\theta\tan\theta$$

$$\sec\left(\tfrac{\pi}{6}\right) - \cos\left(\tfrac{\pi}{6}\right) = \sin\left(\tfrac{\pi}{6}\right)\tan\left(\tfrac{\pi}{6}\right)$$

$$\left(\frac{2}{\sqrt{3}}\right) - \left(\frac{\sqrt{3}}{2}\right) = \left(\frac{1}{2}\right)\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\begin{matrix}\cdot 2\\ \cdot 2\end{matrix}$$

$$\frac{4}{2\sqrt{3}} - \frac{\sqrt{3}\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}\cdot\left(\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\right)$$

$$\frac{4-3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$\frac{1}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

Though the work above shows that both sides of the equation have the same value for one particular value of θ, this is not enough to show that these two expressions are equivalent for ALL values of θ. We must be able to show analytically that these two expressions are equivalent.

To do this, our goal is to rewrite both sides using the basic trigonometric identities to show that each expression can be simplified to the same expression. Often, this means that the left side can be rewritten as the right side, the right side can be rewritten as the left side, or both sides can be rewritten to form separate equivalent expression.

Analytically show that the equations below represent trigonometric identity statements.

1. $sec\,\theta - cos\,\theta = sin\,\theta\,tan\,\theta$

$$\frac{1}{\cos\theta} - \frac{\cos\theta}{1} =$$

$$\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} =$$

$$\frac{1-\cos^2\theta}{\cos\theta} =$$

$$\boxed{\begin{matrix}\sin^2\theta+\cos^2\theta=1\\ \sin^2\theta=1-\cos^2\theta\end{matrix}}$$

$$\frac{\sin^2\theta}{\cos\theta} =$$

$$\sin\theta\cdot\frac{\sin\theta}{\cos\theta} =$$

$$\sin\theta\cdot\tan\theta = \sin\theta\tan\theta$$

2. $\frac{cos\,\theta}{sin\,\theta\,cot\,\theta} = sec\,\theta\,cos\,\theta$

$$\frac{\cos\theta}{\cancel{\sin\theta}\,\frac{\cos\theta}{\cancel{\sin\theta}}} =$$

$$\frac{\cos\theta}{\cos\theta} =$$

$$1 = 1$$

Analytically show that the equations below represent trigonometric identity statements.

3. $\sec\theta\,(\csc\theta - \cot\theta\cos\theta) = \tan\theta$

$\sec\theta\csc\theta - \sec\theta\cot\theta\cos\theta =$

$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} - \frac{\cos\theta\cos\theta}{\sin\theta\cos\theta} =$

$\frac{1-\cos^2\theta}{\sin\theta\cos\theta} =$

$\boxed{\begin{array}{l}\sin^2\theta+\cos^2\theta=1\\ \sin^2\theta=1-\cos^2\theta\end{array}}$

$\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}\cos\theta} =$

$\frac{\sin\theta}{\cos\theta} =$

$\tan\theta = \tan\theta$

4. $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = (\sec^2\theta) - \tan^2\theta$

$\sin\theta\sin\theta + \cos\theta\cos\theta = (1+\tan^2\theta) - \tan^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$1 = 1$

5. $\csc\theta - \sin\theta = \cos\theta\cot\theta$

$\boxed{\begin{array}{l}\sin^2\theta+\cos^2\theta=1\\ \cos^2\theta=1-\sin^2\theta\end{array}}$

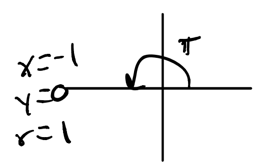
$\frac{1}{\sin\theta} - \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\sin\theta} =$

$\frac{1-\sin^2\theta}{\sin\theta} =$

$\frac{\cos^2\theta}{\sin\theta} =$

$\cos\theta\cdot\frac{\cos\theta}{\sin\theta} =$

$\cos\theta\cdot\cot\theta = \cos\theta\cot\theta$



6. In your work above, you verified that $\csc\theta - \sin\theta = \cos\theta\cot\theta$ is an identity. Explain why $\theta = \pi$ would not be an appropriate value to try to show numerically that this equation is an identity.

$\csc(\pi) - \sin(\pi) = \cos(\pi)\cot(\pi)$

$\frac{1}{0} - 0 = (-1)\left(\frac{-1}{0}\right)$

und $- 0 = (-1)\cdot$ und

$\boxed{\text{when } \theta = \pi, \ \csc\theta \text{ and } \cot\theta \text{ are undefined.}}$

Analytically show that the equations below represent trigonometric identity statements.

7. $\frac{\sin^2 x+\cos^2 x}{\tan x} = \cot x$

$\frac{1}{\tan x} =$

$\cot x = \cot x$

8. $(\sec\theta + 1)(\csc\theta - \cot\theta) = \tan\theta$

$\left(\frac{1}{\cos\theta} + 1\frac{\cos\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) =$

$\left(\frac{1+\cos\theta}{\cos\theta}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right) =$

$\frac{1-\cos^2\theta}{\sin\theta\cos\theta} =$

$\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}\cos\theta} =$

$\frac{\sin\theta}{\cos\theta} =$

$\tan\theta = \tan\theta$

Analytically show that the equations below represent trigonometric identity statements.

9. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta} =$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} =$

$\frac{1}{\sin\theta\cos\theta} =$

$\csc\theta\sec\theta =$

$\sec\theta\csc\theta = \sec\theta\csc\theta$

10. $\frac{(1+\sin\theta)}{(1+\sin\theta)}\cdot\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta}\cdot\frac{(1-\sin\theta)}{(1-\sin\theta)} = 2\tan\theta$

$\frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{1-\sin^2\theta} =$

$\frac{\cos\theta[(1+\sin\theta) - (1-\sin\theta)]}{\cos^2\theta} =$

$\frac{\cos\theta[1+\sin\theta - 1+\sin\theta]}{\cos^2\theta} =$

$\frac{\cancel{\cos\theta}[2\sin\theta]}{\cos^{\cancel{2}}\theta} =$

$2\frac{\sin\theta}{\cos\theta} =$

$2\tan\theta = 2\tan\theta$