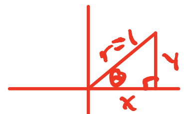
Notes 10.1 Using the Basic Trigonometric Identities

There are seven basic trigonometric identities, four of which you already know. We will just rewrite them in a different form. The other three are what are known as the Pythagorean Identities.

1. $\tan\vartheta = \frac{1}{\cot\theta}$

2. $\tan\theta = \frac{\sin\theta}{\cos\theta}$



$\tan\theta = \frac{\sin\theta = y}{\cos\theta = x}$

2. $\cot\vartheta = \frac{1}{\tan\theta}$

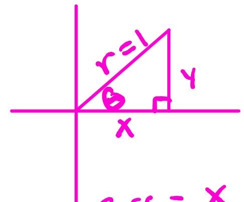
4. $\cot\theta = \frac{\cos\theta}{\sin\theta}$

3. $\sec\vartheta = \frac{1}{\cos\theta} \Longrightarrow \cos\theta = \frac{1}{\sec\theta}$

4. $\csc\vartheta = \frac{1}{\sin\theta} \Longrightarrow \sin\theta = \frac{1}{\csc\theta}$

Pythagorean Identities

5. $\boxed{\cos^2\theta + \sin^2\theta = 1 \quad \sin^2\theta + \cos^2\theta = 1}$



$\cos\theta = x$

$\sin\theta = y$

$x^2 + y^2 = r^2$

$\underbrace{(\cos\theta)}^2 + \underbrace{(\sin\theta)}^2 = (1)^2$

$\cos^2\theta + \sin^2\theta = 1$

6. $1 + \tan^2\theta = \sec^2\theta$ Verify that this is true by starting with the first Pythagorean Identity.

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

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$$\frac{\sin^2\theta}{\cos^2\theta} = \left(\frac{\sin\theta}{\cos\theta}\right)^2 = (\tan\theta)^2 = \tan^2\theta$$

7. $1 + \cot^2\theta = \csc^2\theta$ Verify that this is true by starting with the first Pythagorean Identity.

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

no equal sign

Rewrite each of the following trigonometric expressions in terms of sine and/or cosine and completely simplify.

1. $\sin\theta\,(\csc\theta - \cos\theta\tan\theta)$

$= \sin\theta\left(\frac{1}{\sin\theta} - \cancel{\cos\theta}\cdot\frac{\sin\theta}{\cancel{\cos\theta}}\right)$

$= \sin\theta\left(\frac{1}{\sin\theta} - \sin\theta\right)$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

SIDE WORK

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

2. $\frac{\sec^2 x - \tan^2 x}{\csc x} = \frac{(1+\cancel{\tan^2 x}) - \cancel{\tan^2 x}}{\csc x}$

$= \frac{1}{\csc x}$

$= \sin x$

3. $\cos\theta\,(\tan\theta + \cot\theta)$

$= \cos\theta\cdot\tan\theta + \cos\theta\cdot\cot\theta$

$= \cancel{\cos\theta}\cdot\frac{\sin\theta}{\cancel{\cos\theta}} + \cos\theta\cdot\frac{\cos\theta}{\sin\theta}$

$= \frac{\sin\theta}{1} + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

4. $\frac{\csc\theta}{1+\cot^2\theta} = \frac{\cancel{\csc\theta}}{\csc^{\cancel{2}}\theta}$

$= \frac{1}{\csc\theta}$

$= \sin\theta$

5. $\frac{\tan\alpha\,(\overbrace{1+\cot^2\alpha}^{\csc^2\alpha})}{\cot\alpha} = \frac{\tan\alpha(\csc^2\alpha)}{\cot\alpha}$

$= \tan\alpha\cdot\csc^2\alpha\cdot\tan\alpha$

$= \tan^2\alpha\,\csc^2\alpha$

$= \frac{\cancel{\sin^2\alpha}}{\cos^2\alpha}\cdot\frac{1}{\cancel{\sin^2\alpha}}$

$= \frac{1}{\cos^2\alpha}$

$\frac{\tan\alpha(\csc^2\alpha)}{\cot\alpha}$

$= \frac{\tan\alpha\cdot\csc^2\alpha}{\cancel{\tan\alpha}}\cdot\frac{\tan\alpha}{\cancel{\tan\alpha}}$

$= \tan\alpha\,\csc^2\alpha\,\tan\alpha$

Rewrite each of the following trigonometric expressions in terms of a SINGLE trigonometric ratio.

6. $\sec\theta\csc\theta - \tan\theta$

$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta}$

$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{\cos^{\cancel{2}}\theta}{\sin\theta\cancel{\cos\theta}}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

8. $\frac{\sec x}{\sin x} - \frac{\sec x}{\csc x} = \frac{1}{\sin x\cos x} - \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}$

$= \frac{1}{\sin x\cos x} - \frac{\sin^2 x}{\sin x\cos x}$

$= \frac{1-\sin^2 x}{\sin x\cos x}$

$= \frac{\cos^{\cancel{2}} x}{\sin x\cancel{\cos x}}$

$= \frac{\cos x}{\sin x}$

$= \cot x$

7. $\frac{\sec x+\csc x}{1+\tan x} = \frac{\frac{\sin x}{\sin x}\cdot\frac{1}{\cos x} + \frac{1}{\sin x}\cdot\frac{\cos x}{\cos x}}{\frac{\cos x}{\cos x}\cdot 1 + \frac{\sin x}{\cos x}}$

$= \frac{\frac{\sin x}{\sin x\cos x} + \frac{\cos x}{\sin x\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}$

$= \frac{\frac{\sin x+\cos x}{\sin x\cos x}}{\frac{\cancel{\cos x+\sin x}}{\cancel{\cos x}}} \quad \frac{\cos x}{\cos x+\sin x}$

$= \frac{\cancel{\sin x+\cos x}}{\sin x\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\cancel{\cos x+\sin x}}$

$= \frac{1}{\sin x}$

$= \csc x$

9. $\frac{\sec\theta-\cos\theta}{\sin^2\theta\sec^2\theta} = \frac{\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}}{\sin^2\theta\cdot\frac{1}{\cos^2\theta}}$

$\boxed{\begin{array}{l}\sin^2\theta+\cos^2\theta=1\\ \sin^2\theta=1-\cos^2\theta\end{array}}$

$= \frac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\frac{\sin^2\theta}{\cancel{\cos\theta}}}{\frac{\sin^2\theta}{\cancel{\cos^2\theta}}}\cdot\frac{\cos^2\theta}{\cancel{\cos^2\theta}}$

$= \frac{\cancel{\sin^2\theta}\cdot\cos\theta}{\cancel{\sin^2\theta}}$

$= \cos\theta$

Simplify the following trigonometric expression.

10. $\frac{(\cos\theta-1)}{(\cos\theta-1)}\frac{(1+\sec\theta)}{\sec\theta-1}+\frac{(1+\cos\theta)}{\cos\theta-1}\frac{(\sec\theta-1)}{(\sec\theta-1)}$

$$= \frac{(\cos\theta-1)(1+\sec\theta)}{(\cos\theta-1)(\sec\theta-1)} + \frac{(1+\cos\theta)(\sec\theta-1)}{(\cos\theta-1)(\sec\theta-1)}$$

$$= \frac{(\cancel{\cos\theta} + \cos\theta\sec\theta - 1 - \cancel{\sec\theta}) + (\cancel{\sec\theta} - 1 + \cos\theta\sec\theta - \cancel{\cos\theta})}{\cos\theta\cdot\sec\theta - \cos\theta - \sec\theta + 1}$$

$$= \frac{1 - 1 - 1 + 1}{1 - \cos\theta - \sec\theta + 1}$$

$$= \frac{0}{2 - \sec\theta - \sec\theta}$$

$$= 0$$