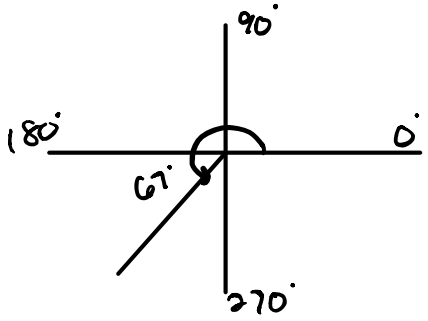
Notes 9.4 Finding Values of Trigonometric Ratios

For each of the angles, θ, described below, draw the angle in standard position, find the measure of the reference angle, a positive co-terminal angle, and a negative co-terminal angle.

1. $\theta = -473°$

Co-term: $-473° + 360° = -113°$

Co-term: $-473° + 2(360°) = 247°$



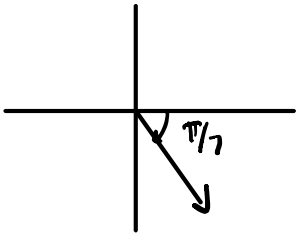
Reference $\angle = 67°$

2. $\theta = \frac{13\pi}{7}$

(2π)

Coterminal $= \frac{13\pi}{7} - \frac{14\pi}{7} = \frac{-\pi}{7}$

Coterminal $= \frac{13\pi}{7} + \frac{14\pi}{7} = \frac{27\pi}{7}$



Reference $\angle = \pi/7$

For each of the angles, θ, described below, determine the quadrant or axis in/on which the terminal side of the angle lies. If no such angle exists, state why. Completely explain your reasoning.

3. $\cos \vartheta < 0$ and $\tan \vartheta < 0$

If $\cos\theta = \frac{x}{r} < 0$, then the angle terminates to the left of the y-axis.

If $\tan\theta = \frac{y}{x} < 0$, then the angle terminates in quad II or IV

$\therefore \theta$ terminates in Quad II

4. $\cot \vartheta$ is undefined and $\cos \vartheta < 0$

If $\cot\theta = \frac{x}{y}$ is undefined, then θ terminates on the x-axis.

If $\cos\theta = \frac{x}{r} < 0$, then the angle terminates to the left of the y-axis.

$\therefore \theta$ terminates on the negative x-axis.

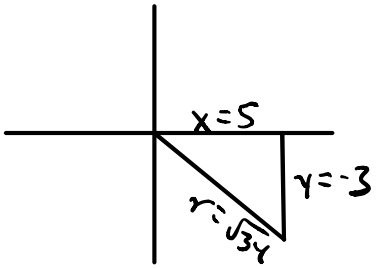
The terminal side of an angle, α, passes through the points given below. Find the exact value of each of the six trigonometric ratios of α.

5. α passes through (5, –3)

$\sin\alpha = \frac{-3}{\sqrt{34}}$ $\quad \csc\alpha = \frac{\sqrt{34}}{-3}$

$\cos\alpha = \frac{5}{\sqrt{34}}$ $\quad \sec\alpha = \frac{\sqrt{34}}{5}$

$\tan\alpha = \frac{-3}{5}$ $\quad \cot\alpha = \frac{5}{-3}$



$x^2 + y^2 = r^2$

$(5)^2 + (-3)^2 = r^2$

$25 + 9 = r^2$

$34 = r^2$

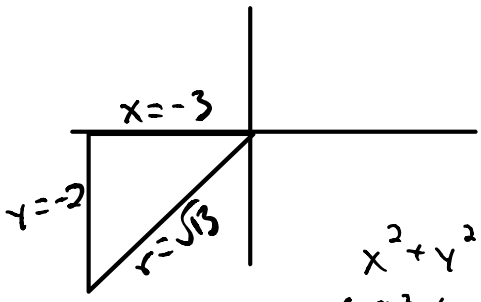
$\pm\sqrt{34} = r$

6. α passes through (–3, –2)

$\sin\alpha = \frac{-2}{\sqrt{13}}$ $\quad \csc\alpha = \frac{\sqrt{13}}{-2}$

$\cos\alpha = \frac{-3}{\sqrt{13}}$ $\quad \sec\alpha = \frac{\sqrt{13}}{-3}$

$\tan\alpha = \frac{2}{3}$ $\quad \cot\alpha = \frac{3}{2}$



$x^2 + y^2 = r^2$

$(-3)^2 + (-2)^2 = r^2$

$9 + 4 = r^2$

$13 = r^2$

$\pm\sqrt{13} = r$

7. Suppose that $\sin\alpha = -\frac{3}{5}$ and $\tan\alpha < 0$. Use this information to answer the following questions.

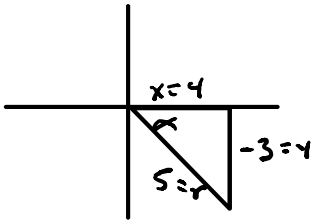
a. In which quadrant does the angle α terminate? Explain your reasoning.

$\sin\alpha = \frac{-3}{5} = \frac{y}{r}$ ∴ α must terminate below the x-axis.

$\tan\alpha = \frac{y}{x} < 0$ ∴ α must terminate in QUAD II or IV.

∴ α terminates in QUAD IV

b. Draw and label the reference triangle for α and find the exact value of the other 5 trigonometric ratios of α.



PT (3-4-5)

$\sin\alpha = \frac{-3}{5}$ $\quad \csc\alpha = \frac{5}{-3}$

$\cos\alpha = \frac{4}{5}$ $\quad \sec\alpha = \frac{5}{4}$

$\tan\alpha = \frac{-3}{4}$ $\quad \cot\alpha = \frac{4}{-3}$

8. Suppose that $sec\ \theta = -\frac{3}{2}$ and $tan\ \theta > 0$. Use this information to answer the following questions.

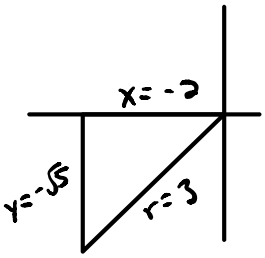
a. In which quadrant does the angle ϑ terminate? Explain your reasoning.

$\sec\theta = \frac{r}{x} = \frac{3}{-2}$ ∴ θ terminates to the left of y-axis.

$\tan\theta = \frac{y}{x} > 0$ ∴ θ terminates in QUAD I or III

∴ θ terminates in QUAD III.

b. Draw and label the reference triangle for ϑ and find the exact value of the other 5 trigonometric ratios of ϑ.



$\sin\alpha = \frac{-\sqrt{5}}{3}$ $\quad \csc\alpha = \frac{3}{-\sqrt{5}}$

$\cos\alpha = \frac{-2}{3}$ $\quad \sec\alpha = \frac{3}{-2}$

$\tan\alpha = \frac{\sqrt{5}}{2}$ $\quad \cot\alpha = \frac{2}{\sqrt{5}}$

$x^2 + y^2 = r^2$

$(-2)^2 + y^2 = (3)^2$

$4 + y^2 = 9$

$y^2 = 5$

$y = \pm\sqrt{5}$

9. Suppose that $csc\ \alpha = \frac{7}{3}$ and $cot\ \alpha < 0$. Use this information to answer the following questions.

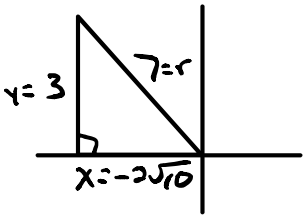
a. In which quadrant does the angle α terminate? Explain your reasoning.

$\csc\alpha = \frac{r}{y} = \frac{7}{3}$ ∴ α terminates above the x-axis.

$\cot\alpha = \frac{x}{y} < 0$ ∴ α terminates in QUAD II or IV

∴ α terminates in QUAD II

b. Draw and label the reference triangle for α and find the exact value of the other 5 trigonometric ratios of α.



$\sin\alpha = \frac{3}{7}$ $\quad \csc\alpha = \frac{7}{3}$

$\cos\alpha = \frac{-2\sqrt{10}}{7}$ $\quad \sec\alpha = \frac{7}{-2\sqrt{10}}$

$\tan\alpha = \frac{3}{-2\sqrt{10}}$ $\quad \cot\alpha = \frac{-2\sqrt{10}}{3}$

$x^2 + y^2 = r^2$

$x^2 + (3)^2 = (7)^2$

$x^2 + 9 = 49$

$x^2 = 40$

$x = \pm\sqrt{40}$

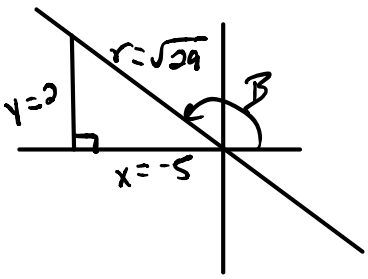
$x = -2\sqrt{10}$

10. The terminal side of an angle β lies in quadrant II along the line $2x + 5y = 0$. Order the values $\sin\beta$, $\tan\beta$, and $\csc\beta$ from least to greatest.

$2x+5y=0$

$5y=-2x$

$y=-\frac{2}{5}x$



$x^2+y^2=r^2$

$(-5)^2+(2)^2=r^2$

$25+4=r^2$

$29=r^2$

$\pm\sqrt{29}=r$

$\sin\beta=\frac{2}{\sqrt{29}}$

$\csc\beta=\frac{\sqrt{29}}{2}$ BIGGEST

$\tan\beta=\frac{2}{-5}$ Smallest

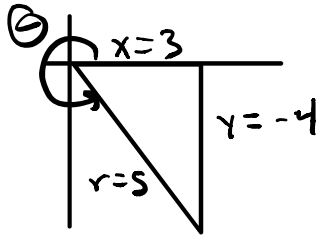
$\tan\beta, \sin\beta, \csc\beta$

11. Suppose $\tan\theta=-\frac{4}{3}$ and $\sec\theta>0$. Find the values of csc ϑ and cos ϑ.

$\tan\theta=\frac{-y}{3}=\frac{y}{x}<0$ ∴ θ terminates in QUAD II and IV

$\sec\theta=\frac{r}{x}>0$ ∴ θ terminates to the right of y-axis.

∴ θ terminates in QUAD IV.



PT(3-4-5)

$\csc\theta=\frac{5}{-4}$

$\cos\theta=\frac{3}{5}$