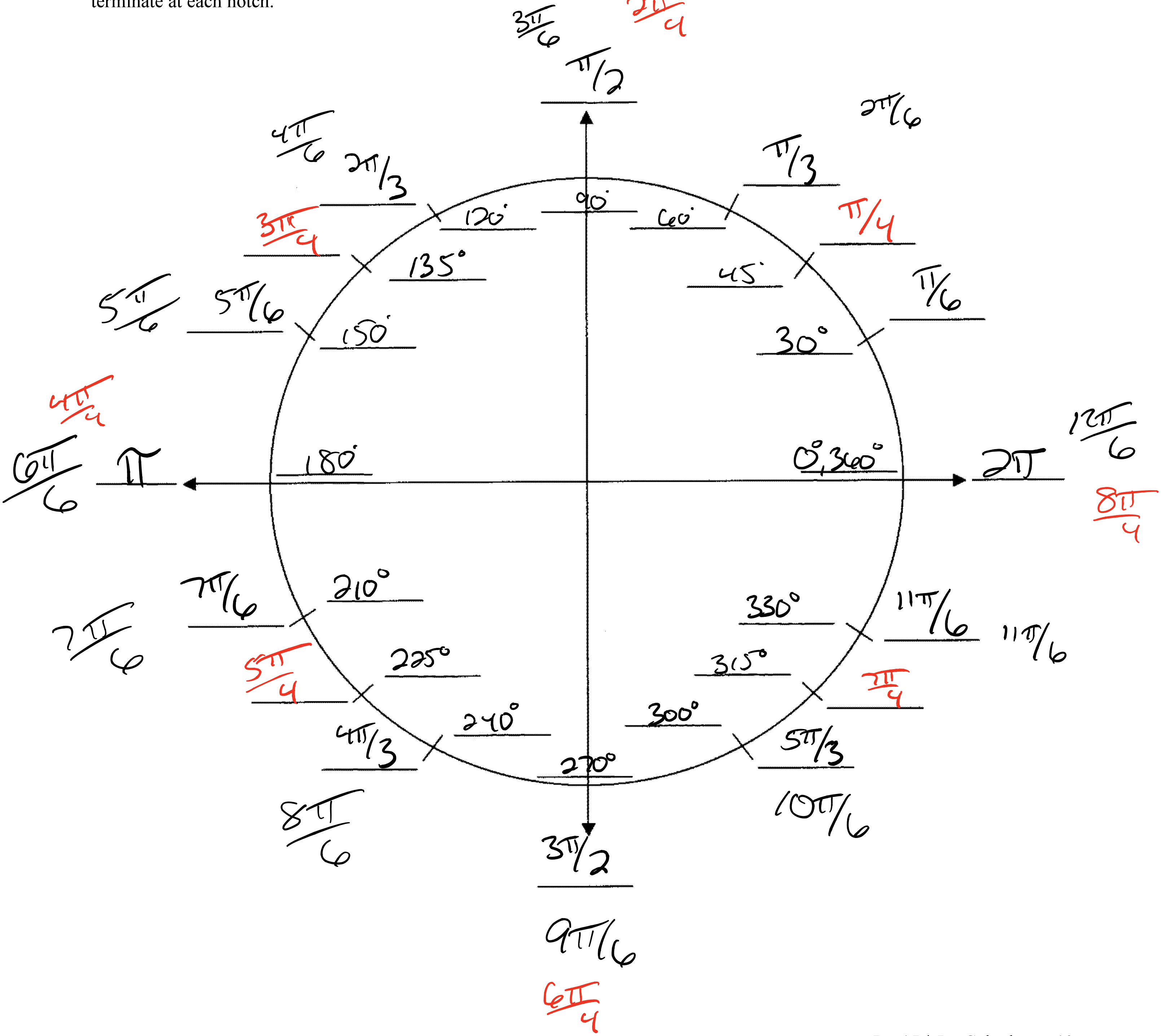
Notes 9.3 Developing and Using the Unit Circle

On the blanks on the inside of the circle, write the degree measure of the angle θ, where $0° < \theta \le 360°$ that would terminate at each notch.

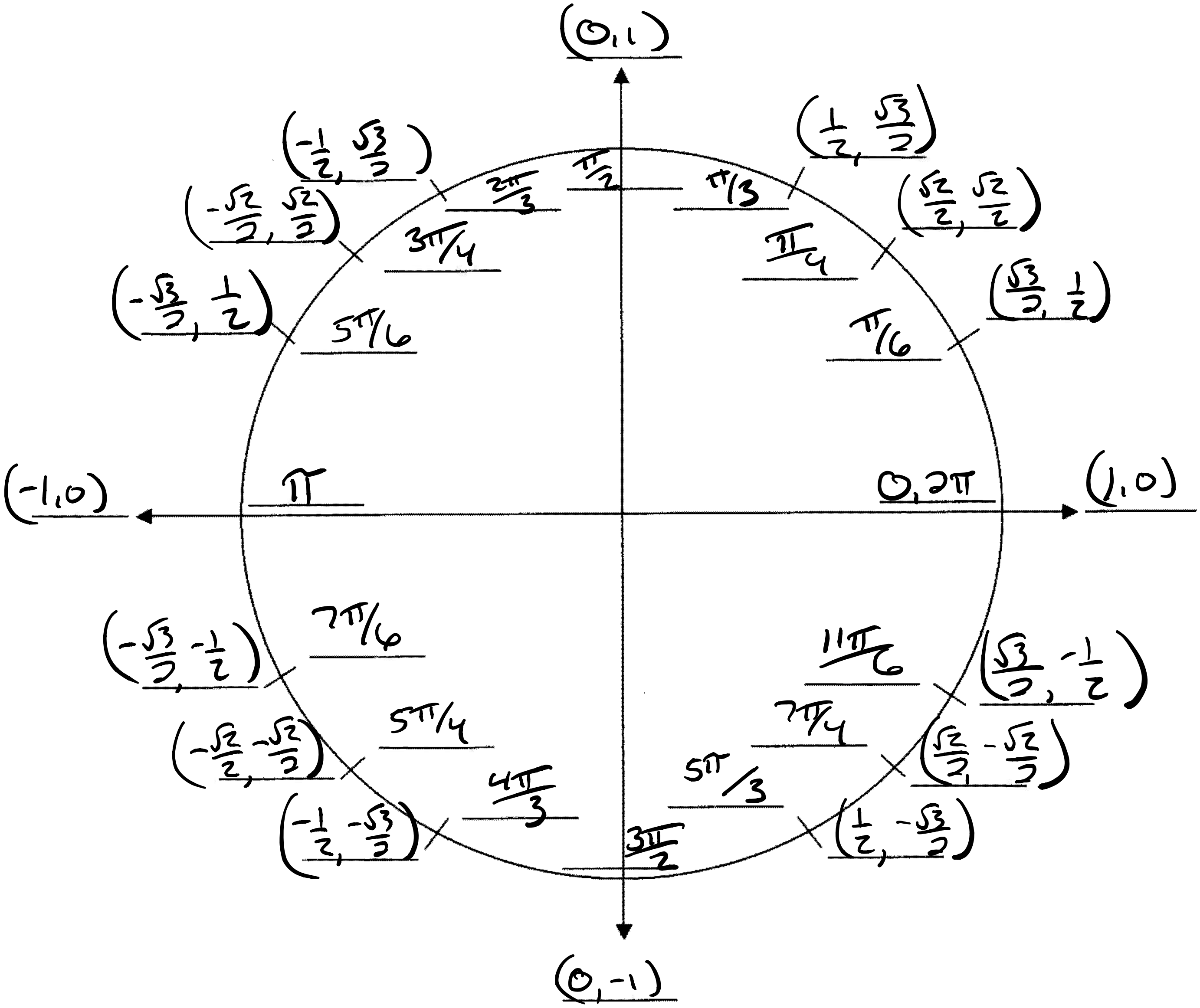
Notice that every angle on the unit circle is a multiple of either 45° or 30°. In the space below, convert these two degree measures into radian measure.

$30° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$ | $45 \cdot \frac{\pi}{180°} = \frac{\pi}{4}$

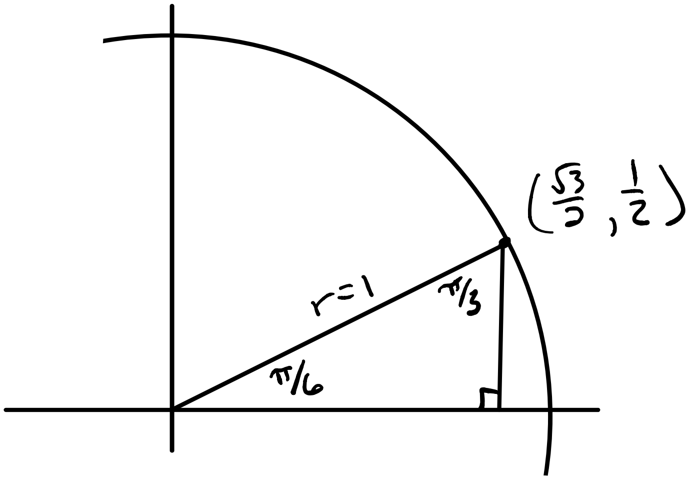
Now, on the blanks on the outside of the circle, write the radian measure of each angle θ, where $0 < \theta \le 2\pi$ that would terminate at each notch.

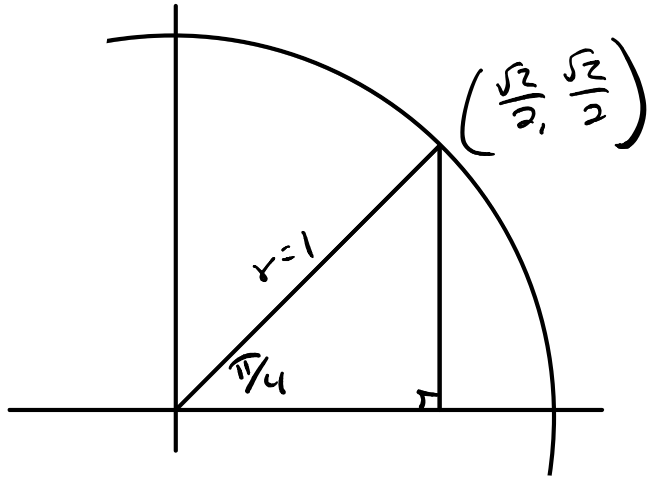


Now, on the blanks on the INSIDE of the circle, write the radian measure of each angle θ, where $0 < \theta \leq 2\pi$ that would terminate at each notch.

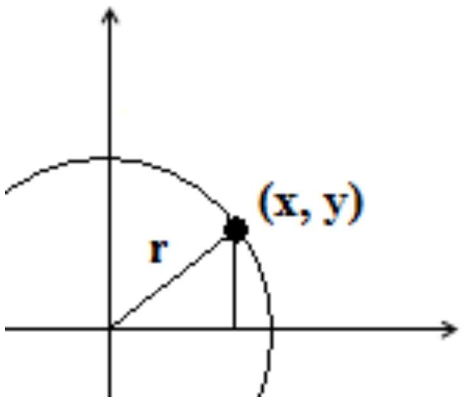


At this point, we want to know what the coordinates of the points on the unit circle are. To develop this, we will use the definition of what the unit circle (a circle whose radius is 1 unit and has a center of (0, 0).





At this point, let's redefine the six trigonometric ratios in terms of circular angle trigonometry.



Right Triangle Definition	New Circular Angle Definition	Newer Unit Circle Definition
$\sin\theta = \dfrac{opposite\ leg}{hypotenuse}$	$\sin\theta = \dfrac{y}{r}$	$\sin\theta = y$
$\cos\theta = \dfrac{adjacent\ leg}{hypotenuse}$	$\cos\theta = \dfrac{x}{r}$	$\cos\theta = x$
$\tan\theta = \dfrac{opposite\ leg}{adjacent\ leg}$	$\tan\theta = \dfrac{y}{x}$	
$\csc\theta = \dfrac{hypotenuse}{opposite\ leg}$	$\csc\theta = \dfrac{r}{y}$	
$\sec\theta = \dfrac{hypotenuse}{adjacent\ leg}$	$\sec\theta = \dfrac{r}{x}$	
$\cot\theta = \dfrac{adjacent\ leg}{opposite\ leg}$	$\cot\theta = \dfrac{x}{y}$	

Using the coordinates on the unit circle, find the exact value of each of the following trigonometric expressions.

$sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$cos\frac{3\pi}{4} + sin\frac{7\pi}{4}$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}$

$= -\frac{2\sqrt{2}}{2}$

$= -\sqrt{2}$

$tan\frac{\pi}{6}$

$= \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

$tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

$sec\frac{11\pi}{6} = \frac{2}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3}$

$csc\frac{4\pi}{2} + tan\frac{5\pi}{4}$

$= csc(2\pi) + tan\left(\frac{5\pi}{4}\right)$

$= \frac{1}{0} + \dfrac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$=$ und $+ 1$

$=$ undefind

x=1, y=0, r=1

$sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$tan\left(-\frac{5\pi}{2}\right)$

$= tan\left(-\frac{\pi}{2}\right)$

$= \frac{-1}{0}$

$=$ undefined

x=0, y=−1, r=1

$-2\,cos(-3\pi) + cot\left(-\frac{\pi}{4}\right)$

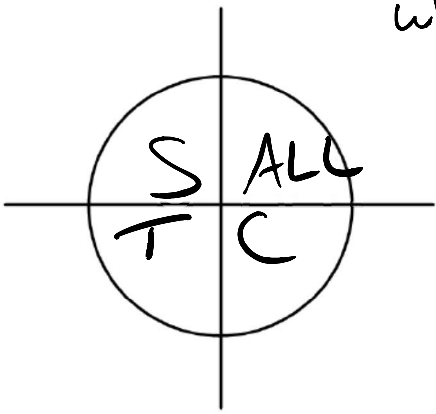
$= -2(-1) + (-1)$

$= 2 - 1$

$= 1$

Positive Negative Quadrants

who's Positive



Described below are angles in standard position. Determine in which quadrant or on what axis each angle terminates. Give an explanation that clearly justifies your reasoning. If no such angle exists, explain why.

a. $cos\,\theta > 0$ and $\tan\theta < 0$

If $\cos\theta = \frac{x}{r} > 0$, then θ terminates to the right of y-axis.

If $\tan\theta = \frac{y}{x} < 0$, then θ terminates in QUAD II or IV

$\therefore$ θ terminates in QUAD IV

b. $cos\,\theta > 0$ and $\tan\theta$ is undefined

If $\cos\theta = \frac{x}{r} > 0$, then θ terminates in QUAD I or IV, or positive x-axis

If $\tan\theta = \frac{y}{x}$ is undefined, then θ terminates on the y-axis.

$\therefore$ θ does not exist

c. $tan\,\theta > 0$ and $\csc\theta < 0$

If $\tan\theta = \frac{y}{x} > 0$, then θ terminates in QUAD I or III

If $\csc\theta = \frac{r}{y} < 0$, then θ terminates below x-axis.

$\therefore$ θ terminates in quad III

d. $\sin\theta < 0$ and $\sec\theta$ is undefined

If $\sin\theta = \frac{y}{r} < 0$, then θ terminates below x-axis.

If $\sec\theta = \frac{r}{x}$ is undefined, then θ terminates on the y-axis.

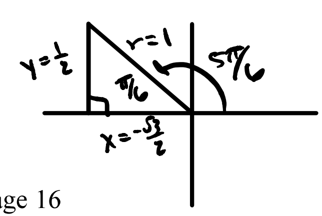
$\therefore$ θ terminates on negative y-axis.

Suppose $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ and $sin\,\theta = \frac{1}{2}$. What is the value of θ and the exact value of the other 5 trigonometric ratios of θ.

- θ terminates to left of y-axis
- $\sin\theta = \frac{1}{2}$

  $\therefore$ θ terminates above x-axis

$\therefore$ θ terminates in QUAD II



$\sin\theta = \frac{1}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\tan\theta = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}}$

$\tan\theta = \frac{-\sqrt{3}}{3}$

$\csc\theta = 2$

$\sec\theta = \frac{-2}{\sqrt{3}}$

$\cot\theta = -\sqrt{3}$