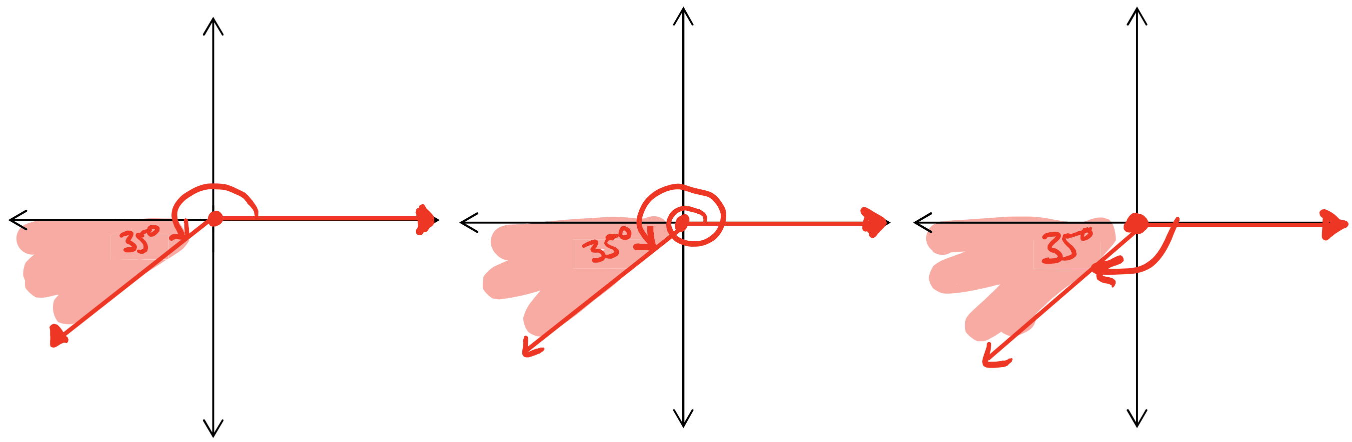
Notes Introduction to Circular Trigonometry and the Unit Circle
A Focus on Terminology

Let's begin by drawing three different circular angles in standard position measuring 215°, 575°, and −145°. As we draw these together, pay close attention to the details that we put on each picture.

215°	575°	−145°



What do all of the angles drawn above have in common with each other?

1) Each angle has the same terminal side.
2) Each angle has the same initial side.
} Coterminal angles.

3) Same reference angle: An acute angle formed by the x-axis and terminal side

What is the one difference that all of the angles drawn above have?

1. Difference in rotation direction
2. Different degrees in standard position
3. Can be different # of complete revolutions.

Based on what you have seen so far, what are the four vital things that all circular angles must have when drawn in standard position.

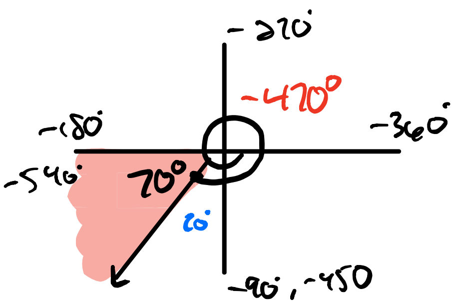
1. Vertex at origin & initial side on positive x-axis
2. Terminal side
3. Arrow of rotation
4. Reference Angle

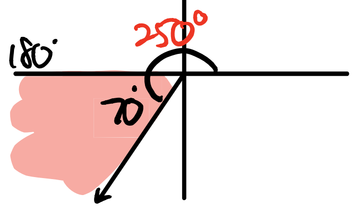
The three angles drawn above are called **coterminal angles**. Give a definition of coterminal angles.

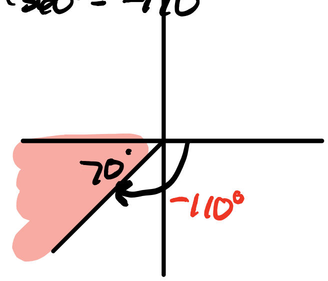
Draw the angle −470° in standard position. Then, find one positive and one negative coterminal angle with −470° and draw them in standard position.

+ coterminal: $-470° + 720° = 250°$

− coterminal: $-470° + 360° = -110°$

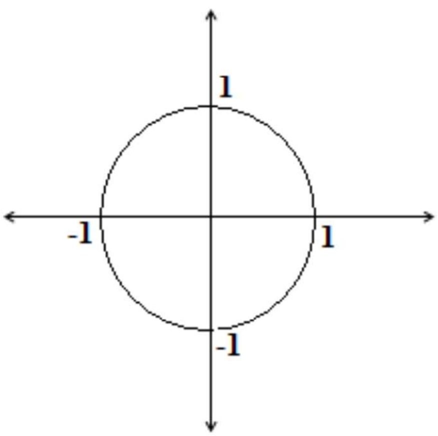






Just as segments can be measured using inches or centimeters, angles can be measured using different units of measurement. In trigonometry, the two units used to measure angles are degrees and radians.

Radian measure is based upon what is called the Unit Circle in trigonometry. By definition, the Unit Circle is a circle whose center is (0,0) and whose radius = 1.



Derivation of Radian Measure Conversion Factor

Find the circumference of the unit circle.

$C = 2\pi r$

$C = 2\pi(1)$

$C = 2\pi$

By what must 360° be multiplied to obtain the circumference of the unit circle?

$360° \cdot \dfrac{\pi \text{ RAD}}{180°} = 2\pi \text{ RAD}$

or

$\cancel{360°} \cdot \dfrac{2\pi}{\cancel{360°}} = 2\pi \text{ RAD}$

To Covert from Degree Measure into Radian Measure	To Covert from Radian Measure into Degree Measure
$\text{Degree} \cdot \dfrac{\pi}{180°} = \text{RAD}$	$\text{RAD} \cdot \dfrac{180°}{\pi} = \text{Degree}$

Given the angle in either degree or radian measure, convert it into the other unit of measure.

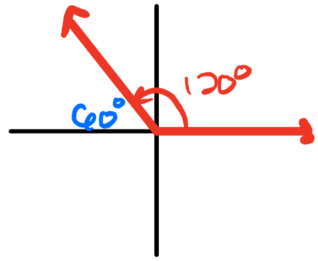
a. 90° $\cancel{90°} \cdot \dfrac{\pi}{\cancel{180°}} = \dfrac{\pi}{2}$	b. −235° Div by 5 $-235° \cdot \dfrac{\pi}{180°} = \dfrac{-235\pi}{180} = \dfrac{-47\pi}{36}$
c. $\dfrac{2\pi}{3}$ $\dfrac{2\cancel{\pi}}{\cancel{3}} \cdot \dfrac{\cancel{180}\ 60°}{\cancel{\pi}} = 120°$	d. $-\dfrac{13\cancel{\pi}}{\cancel{6}} \cdot \dfrac{\cancel{180}\ 30°}{\cancel{\pi}} = -390°$

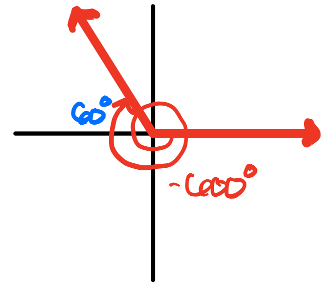
Find one positive and one negative co-terminal angle with $-\dfrac{4\pi}{3}$ and draw them in standard position. Do this by first converting the given angle into degrees.

$-\dfrac{4\cancel{\pi}}{3} \cdot \dfrac{\cancel{180^\circ}\,60^\circ}{\cancel{\pi}} = -240^\circ$

Positive: $-240^\circ + 360^\circ = 120^\circ$

Negative: $-240^\circ - 360^\circ = -600^\circ$



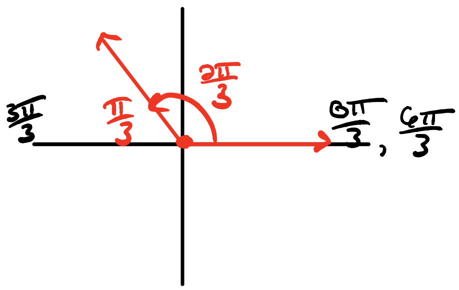


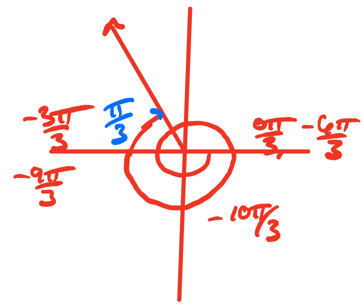
Find one positive and one negative co-terminal angle with $-\dfrac{4\pi}{3}$ and draw them in standard position. Do this without converting the given angle into degrees.

(1 Rev)

Positive: $-\dfrac{4\pi}{3} + \dfrac{6\pi}{3} = \dfrac{2\pi}{3}$

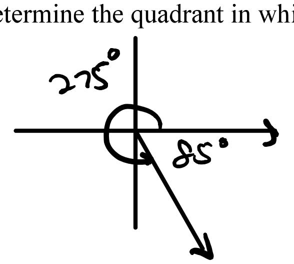
Negative: $-\dfrac{4\pi}{3} - \dfrac{6\pi}{3} = -\dfrac{10\pi}{3}$





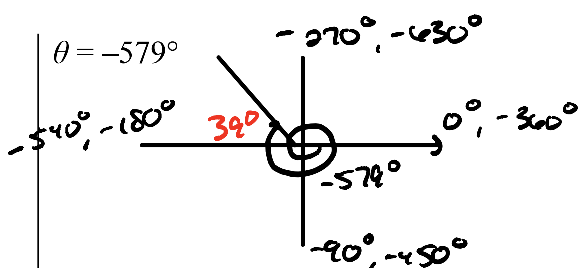
Given the angle, θ, determine the quadrant in which the angle terminates and find the measure of θ', the reference angle.

$\theta = 275^\circ$



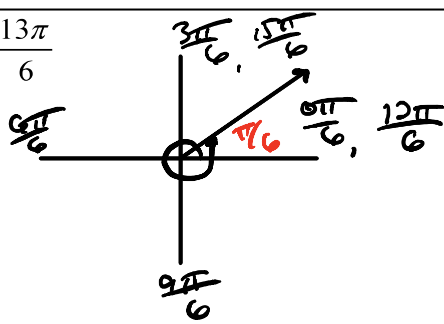
θ terminates in QUAD IV
and $\theta' = 85^\circ$

$\theta = -579^\circ$



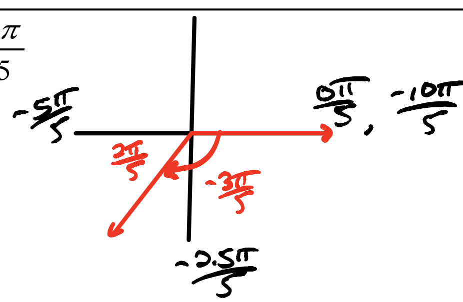
θ terminates in QUAD II
& $\theta' = 39^\circ$

$\theta = \dfrac{13\pi}{6}$



θ terminates in QUAD I
& $\theta' = \dfrac{\pi}{6}$

$\theta = -\dfrac{3\pi}{5}$



θ terminates in QUAD III
and $\theta' = \dfrac{2\pi}{5}$

Extra practice

Convert each of the following degree measures into radians. Leave your answers in terms of π. State in which quadrant or on what axis the terminal side of the angle lies. Show your work.

1. $315° \cdot \frac{\pi}{180°} = \frac{315\pi}{180} = \frac{7\pi}{4}$

   QUAD IV

2. $-135° \cdot \frac{\pi}{180°} = \frac{-135\pi}{180} = \frac{-3\pi}{4}$

   QUAD III

3. $210° \cdot \frac{\pi}{180°} = \frac{21\pi}{18} = \frac{7\pi}{6}$

   QUAD III

4. $-240° \cdot \frac{\pi}{180°} = \frac{-24\pi}{18} = \frac{-4\pi}{3}$

   QUAD II

Convert each of the following radian measures into degrees. Give your answers to the nearest thousandth of a degree, if necessary. State in which quadrant or on what axis the terminal side of the angle lies. Show your work.

5. $\frac{7\cancel{\pi}}{\cancel{6}} \cdot \frac{\overset{30°}{\cancel{180°}}}{\cancel{\pi}} = 210°$

   QUAD III

6. $-3.25 \cdot \frac{180}{\pi}$

   $\approx -186.211°$

   QUAD II

7. $-\frac{13\cancel{\pi}}{\cancel{4}} \cdot \frac{\overset{45°}{\cancel{180°}}}{\cancel{\pi}} = -585°$

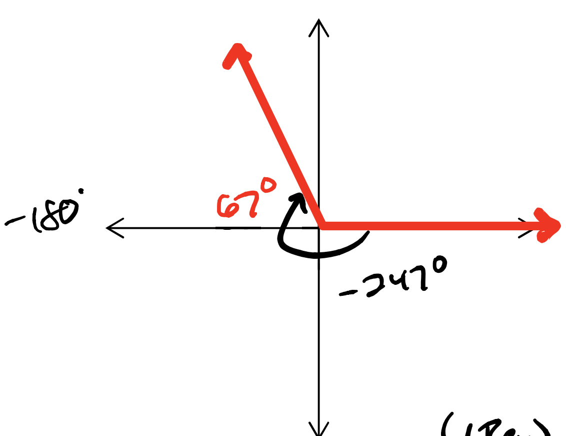
   QUAD II

8. $\frac{9\cancel{\pi}}{\cancel{20}} \cdot \frac{\overset{9°}{\cancel{180°}}}{\cancel{\pi}} = 81°$

   QUAD I

Draw each of the angles below in standard position. Then, find one positive and one negative co-terminal angle. If the angle is given in degrees, then all answers should be in degrees. If the angle is given in radian measure, then all answers should be in radians.

9. $\theta = -247°$



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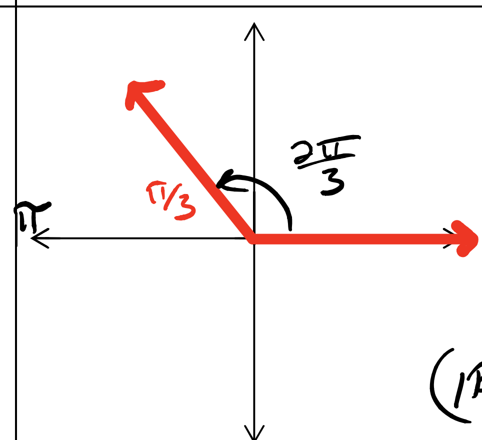
Positive: $-247° + 360° = 113°$

Negative: $-247° - 360° = -607°$

10. $\theta = \frac{2\pi}{3}$



(1 Rev)

Positive: $\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$

Negative: $\frac{2\pi}{3} - \frac{6\pi}{3} = \frac{-4\pi}{3}$