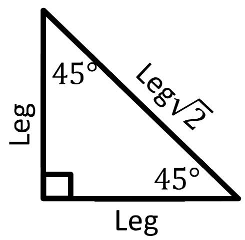
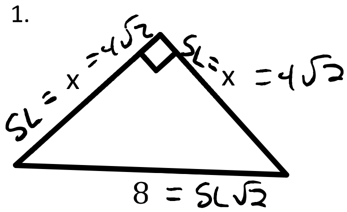
Notes 9.1 Triangle Trigonometry

THE 45° - 45° - 90° TRIANGLE (RIGHT ISOSCELES)



Hypotenuse = Leg$\sqrt{2}$

1.

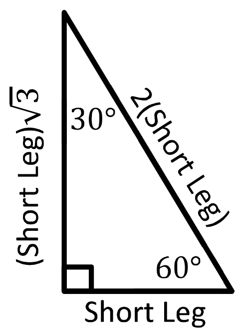


$\frac{8}{\sqrt{2}}$ = SL

$\frac{8\sqrt{2}}{2}$ = SL

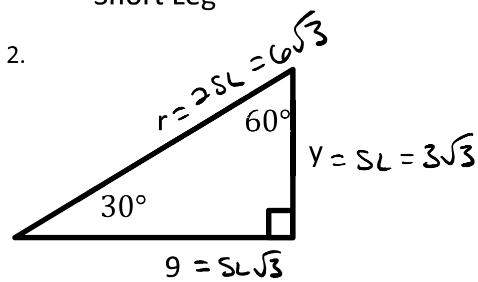
$4\sqrt{2}$ = SL

THE 30° - 60° - 90° TRIANGLE



Long Leg = (Short Leg)$\sqrt{3}$
Hypotenuse = 2(Short Leg)

2.



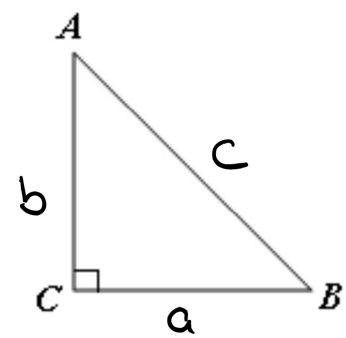
$\frac{9}{\sqrt{3}}$ = SL

$\frac{9\sqrt{3}}{3}$ = SL

$3\sqrt{3}$ = SL

Consider ΔABC shown below to complete the following trigonometric equations

Define each trig ratio.	Find the ratio.	Find the ratio.
$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}}$	$\sin A = \frac{a}{c}$	$\csc A = \frac{c}{a}$
$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$	$\cos A = \frac{b}{c}$	$\sec A = \frac{c}{b}$
$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$	$\tan A = \frac{a}{b}$	$\cot A = \frac{b}{a}$
The secant of an angle, θ, abbreviated $\sec\theta$, is defined to be the reciprocal of $\cos\theta$	$\sin B = \frac{b}{c}$	$\csc B = \frac{c}{b}$
The cosecant of an angle, θ, abbreviated $\csc\theta$, is defined to be the reciprocal of $\sin\theta$	$\cos B = \frac{a}{c}$	$\sec B = \frac{c}{a}$
The cotangent of an angle, θ, abbreviated $\cot\theta$, is defined to be the reciprocal of $\tan\theta$	$\tan B = \frac{b}{a}$	$\cot B = \frac{a}{b}$

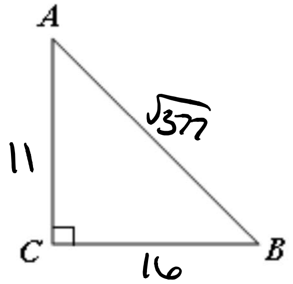


Angles A and B, since their sum is 90° are said to be complementary angles.

What do you notice about the values of the sine and cosine ratios for two complementary angles?

To solve a triangle, we find all missing sides and all missing angles. Use trigonometric ratios to solve each of the following right triangles.

3.



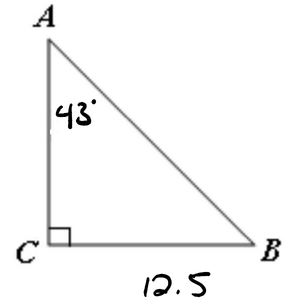
$AC = 11$
$CB = 16$

$$x^2 + y^2 = r^2$$
$$(16)^2 + (11)^2 = AB^2$$
$$256 + 121 = AB^2$$
$$377 = AB^2$$
$$\pm\sqrt{377} = AB$$
$$\sqrt{377} = AB$$

$$\tan(B) = \frac{11}{16}$$
$$B = \tan^{-1}\left(\frac{11}{16}\right)$$
$$B = 34.508$$

$$m\angle A + m\angle B = 90^\circ$$
$$m\angle A + 34.508 = 90^\circ$$
$$m\angle A = 55.491$$

4.



$m\angle A = 43°$
$CB = 12.5$

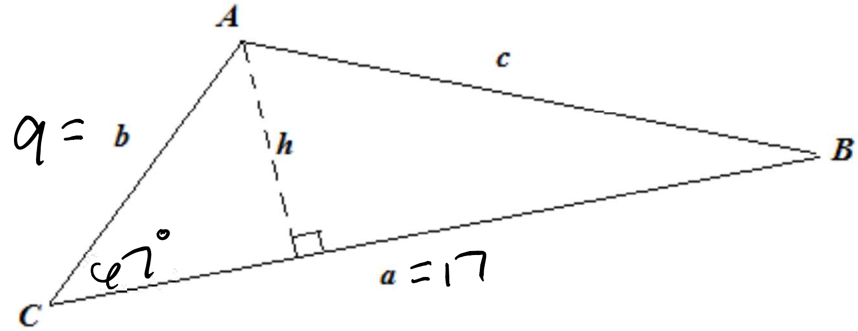
$$m\angle A + m\angle B = 90^\circ$$
$$43^\circ + m\angle B = 90$$
$$m\angle B = 47^\circ$$

$$\sin(43^\circ) = \frac{12.5}{AB}$$
$$AB \sin(43^\circ) = 12.5$$
$$AB = \frac{12.5}{\sin(43^\circ)}$$
$$AB \approx 18.328$$

$$\tan(47^\circ) = \frac{AC}{12.5}$$
$$12.5 \tan(47^\circ) = AC$$
$$13.405 \approx AC$$

Area of Oblique Triangles

Consider the triangle pictured below to answer the questions that follow.



Suppose $b = AC = 9$, $a = CB = 17$, and $m\angle C = 67°$.

Using the formula from geometry, what variable is unknown given this set of information? __h__

Use a trigonometric ratio to find this variable. Show your work below.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(17)\cdot h$

$\sin(67°) = \frac{h}{9}$

$9\sin(67°) = h$

Now, find the area of the triangle.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(17)(9)\sin(67°)$

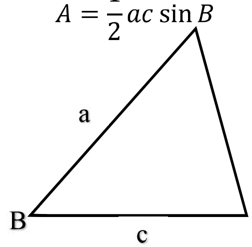
$A \approx 70.419 \text{ un}^2$

Let's derive an alternate formula for the area of a triangle if we are given two sides of the triangle and the angle in between those two sides. Then, test this formula using the given information for the triangle above.

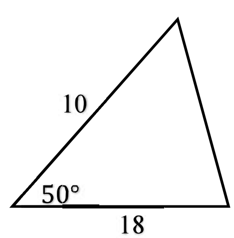
AREA OF A TRIANGLE

GIVEN TWO SIDES AND INCLUDED ANGLE

$$A = \frac{1}{2}ac\sin B$$



5. Find the area of the triangle.



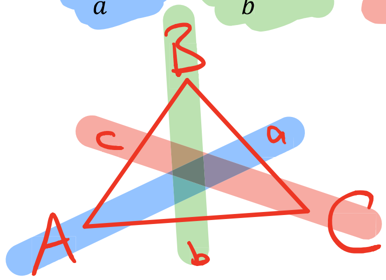
$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(10)(18)\sin(50°)$

$A \approx 68.944 \text{ un}^2$

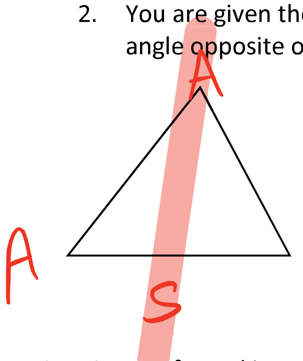
<u>Law of Sines:</u> Let ΔABC be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C respectively. Then,

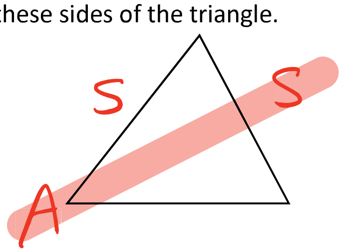
$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle B)}{b} = \frac{\sin(m\angle C)}{c}$$



<u>The Law of Sines can be used to solve a triangle in the following cases:</u>

1. You are given the measure of two angles and any side of a triangle.
2. You are given the measure of two sides and an angle opposite one of these sides of the triangle.



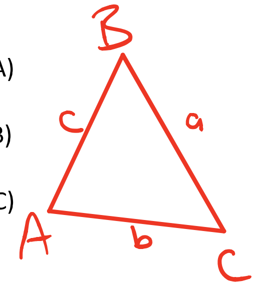


<u>Law of Cosines:</u> Let ΔABC be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C respectively. Then, the following equations hold true.

$a^2 = b^2 + c^2 - 2bc \cos(m\angle A)$

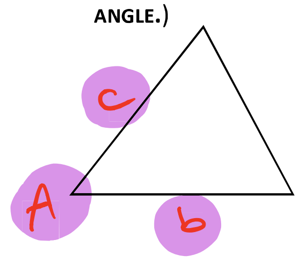
$b^2 = a^2 + c^2 - 2ac \cos(m\angle B)$

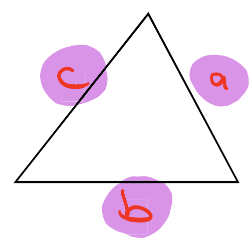
$c^2 = a^2 + b^2 - 2ab \cos(m\angle C)$



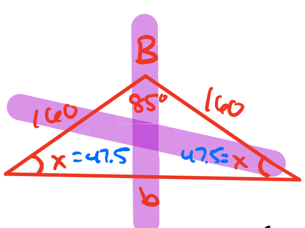
<u>The law of cosines can be used to solve a triangle in the following cases.</u>

1. To find the measure of the third side of any triangle if the measures of the two sides and the included angle are given.
2. To find the measure of an angle of a triangle if the measures of the three sides are given. (If you are given SSS, **You cannot use sines to find the *largest* angle.**)





6. George fenced in a triangular area for his pet parakeet. Two sides of the area are 160 marshmallows long and they meet at an angle of 85°. If a fence is to be built around the area, how many marshmallows of fencing will be needed?



$85 + 2x = 180$

$2x = 95$

$x = 47.5$

$$\frac{\sin(47.5°)}{160} = \frac{\sin(85°)}{b}$$

$b \sin(47.5°) = 160 \sin(85°)$

$$b = \frac{160 \sin(85°)}{\sin(47.5°)}$$

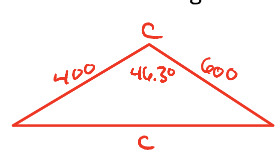
$b \approx 216.2$

$P = 160 + 160 + 216.2$

$P = 536.2$

George needs about 536.2 marshmallows.

7. George fenced in a triangular area for Danny Devito, his miniature pet donkey. Two sides of the area are 400 ears of corn long and 600 ears of corn long and they meet at an angle of 46.3°. If a fence is to be built around the area, how many ears of corn will be needed for the fencing?



$c^2 = a^2 + b^2 - 2ab\cos(m\angle C)$

$c^2 = (400)^2 + (600)^2 - 2(400)(600)\cos(46.3°)$

$c^2 = 160{,}000 + 360{,}000 - 480{,}000\cos(46.3°)$

$c^2 = 520{,}000 - 480{,}000\cos(46.3°)$

$c = \pm\sqrt{520{,}000 - 480{,}000\cos(46.3°)}$

$c \approx 434.0$

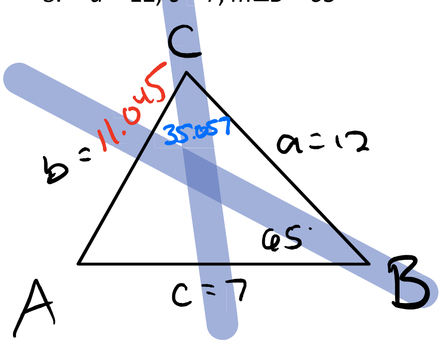
$P = 400 + 600 + 434$

$P = 1434$

George needs 1434 ears of corn.

Given the set of information below, solve each ΔABC described.

8. $a = 12$, $c = 7$, $m\angle B = 65°$



$$b^2 = a^2 + c^2 - 2ac \cdot \cos(m\angle B)$$
$$b^2 = (12)^2 + (7)^2 - 2(12)(7)\cos(65°)$$
$$b^2 = 144 + 49 - 168\cos(65°)$$
$$b^2 = 193 - 168\cos(65°)$$
$$b = \pm\sqrt{193 - 168\cos(65°)}$$
$$b \approx 11.045$$

$$\frac{\sin(65)}{11.045} = \frac{\sin(m\angle C)}{7}$$
$$\frac{7\sin(65°)}{11.045} = \sin(m\angle C)$$
$$\sin^{-1}\left(\frac{7\sin(65)}{11.045}\right) = m\angle C$$
$$35.057 \approx m\angle C$$

$$65° + 35.057 + m\angle A = 180°$$
$$100.057 + m\angle A = 180$$
$$m\angle A = 79.943°$$