Notes 8.4 Graphs and Analysis of Logarithmic Functions
Analytical, Graphical, and Numerical Approaches

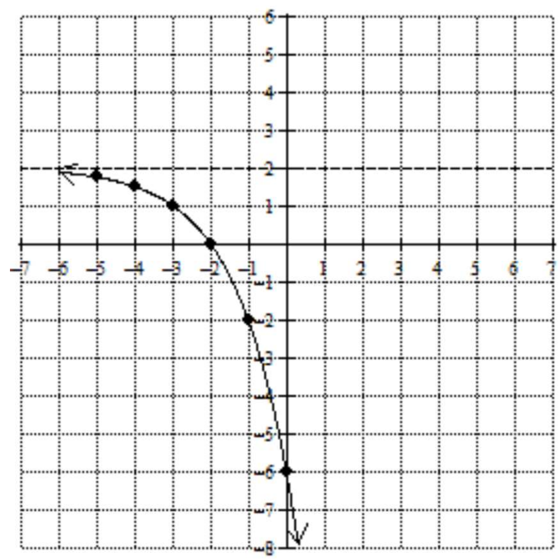
The inverse of an exponential function is a logarithmic function. Let's remember some properties of functions that are "invertible." First, functions must be one-to-one functions in order for their inverse to exist. Summarize how to determine if a function is a one-to-one function or not.

Numerically, a function is 1-1 if no x-values share a y-value.

Graphically, a function is 1-1 if it passes the horizontal line test

Consider for a moment the exponential function $f(x) = -2^{x+3} + 2$, which is graphed to the right. A table of values has been provided to remind you of the transformation of points from the graph of $y = 2^x$.

Points on the graph of $y = 2^x$	Transformed by $(x-3,\ -y+2)$
$\left(-2, \frac{1}{4}\right)$	$\left(-5, 1\frac{3}{4}\right)$
$\left(-1, \frac{1}{2}\right)$	$\left(-4, 1\frac{1}{2}\right)$
$(0,1)$	$(-3,1)$
$(1,2)$	$(-2,0)$
$(2,4)$	$(-1,-2)$
$(3,8)$	$(0,-6)$



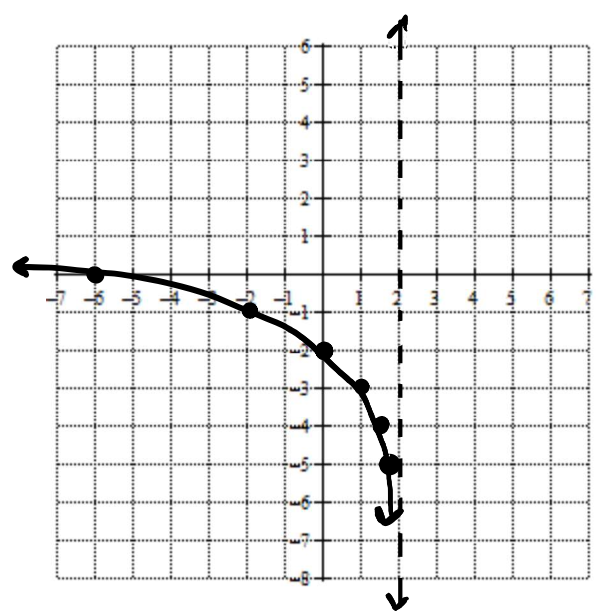
To find the inverse of a function, what is done numerically? Switch x & y-values

The graph of an exponential function has a horizontal asymptote. When the inverse is formed, how do you think this appears in the inverse of the function?

A horizontal asymptote reflected over y=x becomes vertical asymptote

For the exponential function, $f(x)$, above, make a table of values for the inverse function and graph it on the axes to the right.

Points on the graph of $f(x)$	Points on the graph of $f^{-1}(x)$
$\left(-5, 1\frac{3}{4}\right)$	$(1\frac{3}{4}, -5)$
$\left(-4, 1\frac{1}{2}\right)$	$(1\frac{1}{2}, -4)$
$(-3,1)$	$(1,-3)$
$(-2,0)$	$(0,-2)$
$(-1,-2)$	$(-2,-1)$
$(0,-6)$	$(-6,0)$



EG

Now, let's find the equation of the inverse of $f(x)$. Remember, to analytically find the inverse, what do we do to the equation?

1. Switch x & y
2. Solve for y & use $f^{-1}(x)$ notation

Perform the process you just described to the exponential function, $f(x) = -2^{x+3} + 2$, to find the equation of the inverse function, $f^{-1}(x)$.

$$x = -2^{y+3} + 2$$

$$x - 2 = -2^{y+3}$$

$$-(x-2) = 2^{y+3} \quad \text{(Exp form)}$$

$$\log_2[-(x-2)] = y+3 \quad \text{(Log Form)}$$

$$\log_2[-(x-2)] - 3 = y$$

$$f^{-1}(x) = \log_2[-(x-2)] - 3$$

For each of the functions below, find the equation of the inverse function.

1. $f(x) = 2^{-x+3} - 3$	2. $g(x) = 3 + e^{x+2}$	3. $h(x) = -3^{x-2} + 1$
$x = 2^{-y+3} - 3$ $x + 3 = 2^{-y+3}$ (Exp form) $\log_2(x+3) = -y + 3$ (Log Form) $-3 + \log_2(x+3) = -y$ $3 - \log_2(x+3) = y$ $f^{-1}(x) = 3 - \log_2(x+3)$	$x = 3 + e^{y+2}$ $x - 3 = e^{y+2}$ (Exp form) $\ln(x-3) = y + 2$ (Log Form) $\ln(x-3) - 2 = y$ $g^{-1}(x) = \ln(x-3) - 2$	$x = -3^{y-2} + 1$ $x - 1 = -3^{y-2}$ $-(x-1) = 3^{y-2}$ (Exp form) $\log_3[-(x-1)] = y - 2$ (Log Form) $\log_3[-(x-1)] + 2 = y$ $h^{-1}(x) = \log_3[-(x-1)] + 2$

Complete the table below of information about the exponential functions and their inverse logarithmic functions.

Exponential Function	**Domain, Range, and HA of function**	**Domain, Range, & VA of inverse**	**EQ of Inverse Function**	**Set argument of inverse > 0 & solve for *x***
above $f(x) = 2^{-x+3} - 3$	D: $(-\infty, \infty)$ R: $(-3, \infty)$ HA: $y = -3$	D: $(-3, \infty)$ R: $(-\infty, \infty)$ VA: $x = -3$	$f^{-1}(x) = 3 - \log_2(x+3)$	$x + 3 > 0$ $x > -3$ $(-3, \infty)$
above $g(x) = 3 + e^{x+2}$	D: $(-\infty, \infty)$ R: $(3, \infty)$ HA: $y = 3$	D: $(3, \infty)$ R: $(-\infty, \infty)$ VA: $x = 3$	$g^{-1}(x) = \ln(x-3) - 2$	$x + 2 > 0$ $x > -2$ $(-2, \infty)$
below $h(x) = -3^{x-2} + 1$	D: $(-\infty, \infty)$ R: $(-\infty, 1)$ HA: $y = 1$	D: $(-\infty, 1)$ R: $(-\infty, \infty)$ VA: $x = 1$	$h^{-1}(x) = \log_3[-(x-1)] + 2$	$-x + 1 > 0$ $-x > -1$ $x < 1$ $(-\infty, 1)$

Based on the information in the table on the previous page, what inferences can you make regarding how to analytically find the following characteristics of a logarithmic function given the equation of the function?

How to Find the Equation of the Vertical Asymptote of a Logarithmic Function	Solve argument $= 0$
How to Determine the Domain of a Logarithmic Function	Solve argument > 0
How to Determine the Range of a Logarithmic Function	Range always $(-\infty, \infty)$

Find the indicated properties of each of the following logarithmic functions.

1. $F(x) = -2\log_2(2 - x)$	2. $H(x) = -3 + \ln(2x - 3)$
Equation of Vertical Asymptote $2 - x = 0$ $2 = x$	Equation of Vertical Asymptote $2x - 3 = 0$ $2x = 3$ $x = 3/2$
Domain $2 - x > 0$ $2 > x$ $(-\infty, 2)$	Domain $2x - 3 > 0$ $2x > 3$ $x > 3/2$ $(3/2, \infty)$
Range $(-\infty, \infty)$	Range $(-\infty, \infty)$
Does the graph lie to the left or to the right of the vertical asymptote? The graph of $F(x)$ lies to the left of the VA b/c the domain is $(-\infty, 2)$	Does the graph lie to the left or to the right of the vertical asymptote? The graph of $H(x)$ lies to the right of the VA b/c the domain is $(3/2, \infty)$

The table pictured below is a table of values representing an exponential function, $f(x) = e^{x-2} - 3$. Use the table of values to answer the following questions.

x	–5	–2	0	2	4	6
$f(x)$	–2.999	–2.982	–2.865	–2	4.389	51.598

a. What is the equation of the horizontal asymptote of the function, $f(x)$? Explain how you know.

$\lim_{x \to -\infty} f(x) = -3$

$\therefore$ HA $y = -3$

b. Is the graph of $f(x)$ above or below the horizontal asymptote? When the inverse function, $f^{-1}(x)$, is graphed, will the graph be to the right or left of the vertical asymptote? Explain your reasoning.

For $f(x)$, all $y > -3$

$\therefore$ the graph of $f(x)$ is above the HA, $y = -3$

For $f^{-1}(x)$, all $x > -3$

$\therefore$ the graph of $f^{-1}(x)$ is to the right of VA, $x = -3$

c. What is the equation of the vertical asymptote of $f^{-1}(x)$? Explain your reasoning.

For $f(x)$, the HA is $y = -3$

$\therefore$ For $f^{-1}(x)$ the VA is $x = -3$

d. Identify the domain and range of the function $f(x)$ and its inverse, $f^{-1}(x)$.

$f(x)$	$f^{-1}(x)$
D: $(-\infty, \infty)$	D: $(-3, \infty)$
R: $(-3, \infty)$	R: $(-\infty, \infty)$

e. Find the equation of the inverse function, $f^{-1}(x)$. Show your work?

$x = e^{y-2} - 3$

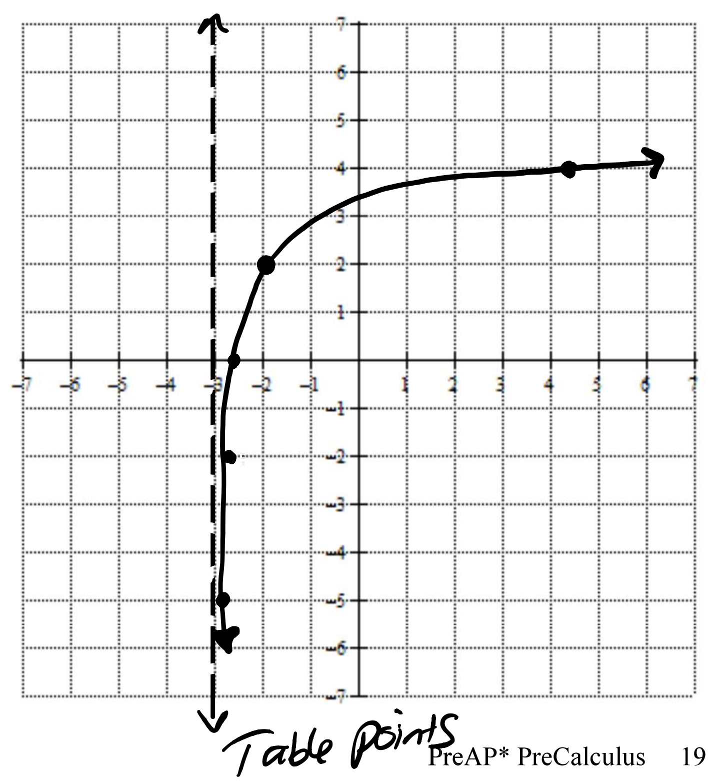
$x + 3 = e^{y-2}$

$\ln(x+3) = y - 2$

$2 + \ln(x+3) = y$

$f^{-1}(x) = \ln(x+3) + 2$

f. On the grid to the right, sketch a graph of $f^{-1}(x)$.



The table pictured below is a table of values representing an exponential function, $f(x) = -2^{x+2} + 3$. Use the table of values to answer the following questions.

x	–7	–4	–1	2	4	7
$f(x)$	2.969	2.75	1	–13	–61	–509

a. What is the equation of the horizontal asymptote of the function, $f(x)$? Explain how you know.

$\lim_{x \to -\infty} f(x) = 3$

$\therefore$ HA $y = 3$

b. Is the graph of $f(x)$ above or below the horizontal asymptote? When the inverse function, $f^{-1}(x)$, is graphed, will the graph be to the right or left of the vertical asymptote? Explain your reasoning.

For $f(x)$, all $y < 3$

$\therefore$ the graph of $f(x)$ is below the HA, $y = 3$

For $f^{-1}(x)$, all $x < 3$

$\therefore$ the graph of $f^{-1}(x)$ is to the left of VA, $x = 3$

c. What is the equation of the vertical asymptote of $f^{-1}(x)$? Explain your reasoning.

For $f(x)$, the HA is $y = 3$

$\therefore$ For $f^{-1}(x)$ the VA is $x = 3$

d. Identify the domain and range of the function $f(x)$ and its inverse, $f^{-1}(x)$.

$f(x)$	$f^{-1}(x)$
D: $(-\infty, \infty)$	D: $(-\infty, 3)$
R: $(-\infty, 3)$	R: $(-\infty, \infty)$

e. Find the equation of the inverse function, $f^{-1}(x)$. Show your work.

$x = -2^{y+2} + 3$

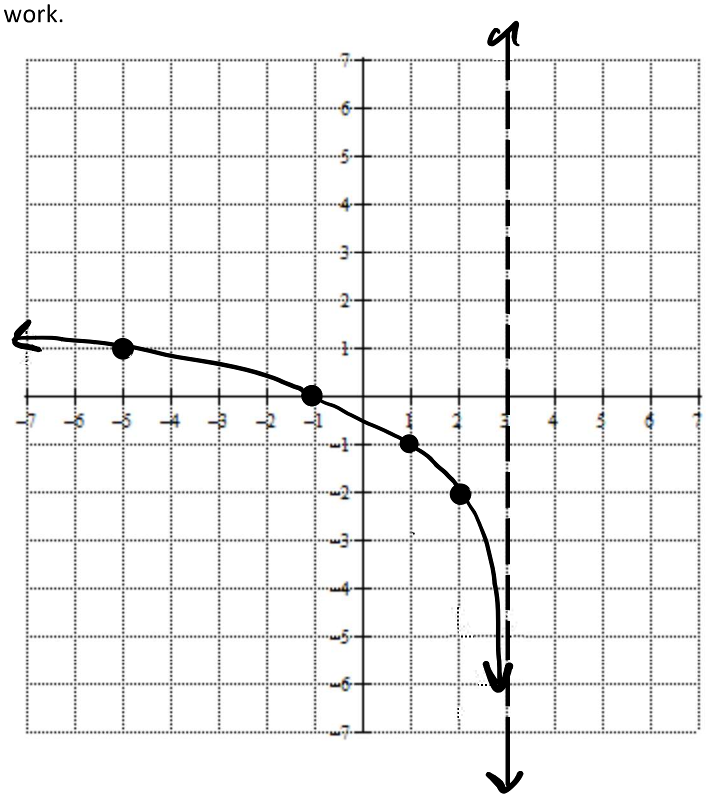
$x - 3 = -2^{y+2}$

$-(x-3) = 2^{y+2}$

$\log_2[-(x-3)] = y + 2$

$\log_2[-(x-3)] - 2 = y$

$f^{-1}(x) = \log_2[-(x-3)] - 2$



f. On the grid to the right, sketch a graph of $f^{-1}(x)$.