Notes 8.3 Solving Exponential and Logarithmic Equations
Using the Logarithms Identities

Log Identity 1	$\log_b b^n =$ n $\ln e^n =$ n	
Log Identity 2	$b^{\log_b a} =$ a $e^{\ln a} =$ a	

Use your calculator to find the value of each of the following expressions.

$5^{\log_5 3} =$ 3	$\log_4 4^7 =$ 7	$\ln e^3 =$ 3	$e^{\ln 4} =$ 4
$2^{\log_2 9} =$ 9	$\log_5 5^{\frac{3}{2}} =$ 3/2	$\ln e^{0.005} =$ 0.005	$e^{\ln\left(\frac{5}{4}\right)} =$ 5/4

Using logarithmic identities, simplify each expression below.

$\ln e^{3x}$ $= 3x$	$\ln \sqrt[3]{e^2}$ $= \ln e^{2/3}$ $= 2/3$	$10^{2\log x}$ $= 10^{\log x^2}$ $= x^2$	$e^{2\ln(x+1)}$ $= e^{\ln(x+1)^2}$ $= (x+1)^2$
$e^{\ln(x+2)}$ $= x+2$	$e^{\ln x+2}$ $= e^{\ln x} \cdot e^2$ $= x \cdot e^2$	$5^{2+\log_5 x}$ $= 5^2 \cdot 5^{\log_5 x}$ $= 25 \cdot x$	$\log 10^{3x} + \ln e^3$ $= 3x + 3$

The processes for solving exponential and logarithmic equations are very similar.

If an exponential equation has only one exponential expression…

① ISO the exponential expression

② Change to Log Form

③ Solve the EQ

Solve: $3 + 2^{3-2x} = 11$

$2^{3-2x} = 8$

$\log_2 8 = 3 - 2x$

$\log_2 2^3 = 3 - 2x$

$3 = 3 - 2x$

$0 = -2x$

$0 = x$

If an exponential equation has exponential expressions on both sides of the equation…

Attempt to…

① Get same bases for each term

② FACTOR

③ If each side has one term, Log Both Sides

If none of the above can be done…

$Y_1 = \text{left}$ use graphing Calc

$Y_2 = \text{right}$

Solve each of the following exponential equations. Round your answers to three decimal places, if necessary.

$3e^{2x+1} = 12$

(Exp form) $e^{2x+1} = 4$

(Log Form) $\ln 4 = 2x + 1$

$-1 + \ln 4 = 2x$

$\frac{1}{2}(-1 + \ln 4) = x$

$0.193 \approx x$

$2^{3-x} + 4 = 10$

(Exp form) $2^{3-x} = 6$

(Log Form) $\log_2 6 = 3 - x$

$-3 + \log_2 6 = -x$

$3 - \log_2 6 = x$

$0.415 \approx x$

$3e^{x-1} + 1 = 10$

$3e^{x-1} = 9$

(Exp form) $e^{x-1} = 3$

(Log Form) $\ln 3 = x - 1$

$1 + \ln 3 = x$

$2.099 \approx x$

$8^{x-1} = 2^{3x+4}$

$(2^3)^{x-1} = 2^{3x+4}$

$2^{3x-3} = 2^{3x+4}$

$3x - 3 = 3x + 4$

$-3 \neq 4$

No solution

FACTOR?

$M = -5e^{2x}$

$A = -4e^{x}$

$N = -5e^{x}, e^{x}$

$e^{2x} - 4e^{x} - 5 = 0$

$\underbrace{e^{2x} - 5e^{x}} + \underbrace{e^{x} - 5} = 0$

$e^{x}(e^{x}-5) + 1(e^{x}-5) = 0$

$(e^{x}-5)(e^{x}+1) = 0$

$e^{x} - 5 = 0$

$e^{x} = 5$

$\ln 5 = x$

$e^{x} + 1 = 0$

$e^{x} = -1$

No Solution

why?

$\ln -1 = x$

Arguments > 0

$2^{x+2} = 5^{x-3}$

$\log 2^{x+2} = \log 5^{x-3}$ (Common Log both sides)

$(x+2)\log 2 = (x-3)\log 5$

$x\log 2 + 2\log 2 = x\log 5 - 3\log 5$

$x\log 2 + \log 4 = x\log 5 - \log 125$

$x\log 2 - x\log 5 = -\log 4 - \log 125$

$x(\log 2 - \log 5) = -1(\log 4 + \log 125)$

$x = \dfrac{-1[\log(4 \cdot 125)]}{\log 2 - \log 5}$

$x = -\dfrac{\log 500}{\log\left(\frac{2}{5}\right)}$

$x = 6.784$

Similarly, we will solve logarithmic equations.

If a logarithm equation has only one logarithm expression…

1. ISO the log expression
2. change to Exponential Form
3. Solve the EQ

If a logarithm equation has logarithm expressions on both sides of the equation…

1. Rewrite each side as a Single Log
2. change to Exponential Form
3. Solve the EQ

If none of the above can be done…

Y_1 = left   use graphing Calc

Y_2 = right

In the end, always check…

to make sure any solution allows the argument to be positive.

$x > -2 \quad x > 0$

Solve: $\ln(x + 2) + \ln x = \ln 8$

$\ln\{x(x+2)\} = \ln 8$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x \neq -4, \; x = 2$

Solve each of the following logarithm equations. Round your answers to three decimal places, if necessary.

$x > -2 \quad x > 1$

$\log_2(x+2) - \log_2(x-1) = 3$

(Log Form) $\log_2\left(\frac{x+2}{x-1}\right) = 3$

(Exp form) $2^3 = \frac{x+2}{x-1}$

$8 = \frac{x+2}{x-1}$

$8x - 8 = x + 2$

$7x - 8 = 2$

$7x = 10$

$x = \frac{10}{7}$

$x > -3$

$\ln(x+3) = \ln 2$

(Property of EQ) $x + 3 = 2$

$x = -1$

$x > \frac{1}{2}$

$-3 + \log_3(2x-1) = 1$

$\log_3(2x-1) = 4$ (Log Form)

$3^4 = 2x - 1$ (Exp form)

$81 = 2x - 1$

$82 = 2x$

$41 = x$

$\log(x+3) + \log x = \log 10$

$\log[x(x+3)] = \log 10$

(Property of EQ) $x^2 + 3x = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5, x = 2$

$x > 3 \quad x > -1$

$\log_2(x-3) - \log_2(x+1) = \log_2 3 - \log_2 7$

$\log_2\left(\frac{x-3}{x+1}\right) = \log_2\left(\frac{3}{7}\right)$

$7\cancel{(x+1)} \frac{x-3}{\cancel{x+1}} = \frac{3}{\cancel{7}} \cancel{7}(x+1)$ (Property of EQ)

$7x - 21 = 3x + 3$

$4x - 21 = 3$

$4x = 24$

$x = 6$

$-3 + \ln(2x-1) = -1$

$\ln(2x-1) = 2$ (Log Form)

$e^2 = 2x - 1$ (Exp form)

$e^2 + 1 = 2x$

$\frac{1}{2}(e^2+1) = x$

$4.195 \approx x$