

Notes 8.2 Properties of Logarithms

<p>Product Property of Logarithms</p>	$\log_b M + \log_b N = \log_b (MN)$ $\ln a + \ln b = \ln(ab)$	<p>When adding logs of the same base, you can write a single log of that base with arguments multiplied.</p>
<p>Quotient Property of Logarithms</p>	$\log_b M - \log_b N = \log_b \left(\frac{M}{N}\right)$ $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$	<p>When subtracting logs of the same base, you can write a single log of that base with arguments divided.</p>
<p>Property of Equality for Logarithms</p>	<p>If $\log_b m = \log_b n$, then <u>$m=n$</u></p>	<p>If two logs of the same base are equal, then their arguments are equal!</p>
<p>Power Property of Logarithms</p>	$\log_b a^n = n \log_b a$ $\ln a^n = n \ln a$	<p>The exponent of an argument can be written as a coefficient of the log</p>

Only use your calculator when needed. Find the value of each of the following logarithm expressions.

$\log_3 9 + \log_3 3 = \underline{2+1=3}$
 $\log_3 27 = \underline{3}$

$\log_2 32 - \log_2 4 = \underline{6-2=4}$
 $\log_2 8 = \underline{4}$

$\log_2 8 + \log_2 4 = \underline{3+2=5}$
 $\log_2 32 = \underline{5}$

$\log_3 81 - \log_3 9 = \underline{4-2=2}$
 $\log_3 9 = \underline{2}$

Using your calculator, find the values of each of the following logarithm expressions.

$$\log_2 5^2 = \underline{4.644}$$

$$2 \cdot \log_2 5 = \underline{4.644}$$

$$\ln 5^2 = \underline{3.219}$$

$$2 \cdot \ln 5 = \underline{3.219}$$

Rewrite each expression as a rational number or a single logarithm of the same base.

$$1. \log_2 6 + \log_2 \frac{8}{3}$$

$$= \log_2 \left(6 \cdot \frac{8}{3} \right)$$

$$= \log_2 (16)$$

$$= 4$$

$$2. \frac{1}{3} \log_3 x - 2 \log_3 y$$

$$= \log_3 x^{1/3} - \log_3 y^2$$

$$= \log_3 \left(\frac{x^{1/3}}{y^2} \right)$$

$$3. \ln(x+2) + \ln x - \ln x^3$$

$$= \ln \left(\frac{(x+2)x}{x^3} \right)$$

$$= \ln \left(\frac{x+2}{x^2} \right)$$

$$4. \log_3 x + 2 \log_3 3 - \log_3 y$$

$$= \log_3 x + \log_3 3^2 - \log_3 y$$

$$= \log_3 \left(\frac{9x}{y} \right)$$

$$5. 3 \log x + 2 \log x - \frac{1}{2} \log x$$

$$= \log x^3 + \log x^2 - \log x^{1/2}$$

$$= \log \frac{x^3 \cdot x^2}{x^{1/2}}$$

$$= \log \frac{x^5}{x^{1/2}}$$

$$= \log \frac{x^{9/2}}{x^{1/2}}$$

$$= \log (x^{9/2})$$

$$6. \ln(x-3) + \ln(2x) - 3 \ln x$$

$$= \ln(x-3) + \ln(2x) - \ln x^3$$

$$= \ln \frac{(x-3)2x}{x^3}$$

$$= \ln \frac{2(x-3)}{x^2}$$

$$= \ln \left(\frac{2x-6}{x^2} \right)$$

$$7. \frac{1}{2} \ln x - 2 \ln y - \ln z$$

$$= \ln \sqrt{x} - \ln y^2 - \ln z$$

$$= \ln \left(\frac{\sqrt{x}}{y^2 z} \right)$$

Rewrite each of the following logarithmic expressions as multiple logarithms in expanded form.

1. $\log_3(xy^2)$

$$= \log_3 x + \log_3 y^2$$

$$= \log_3 x + 2\log_3 y$$

2. $\ln(x\sqrt{y}) = \ln x + \ln \sqrt{y}$

$$= \ln x + \frac{1}{2}\ln y$$

3. $\log_2\left(\frac{x^3}{y^2}\right) = \log_2(x^3) - \log_2 y^2$

$$= 3\log_2 x - 2\log_2 y$$

4. $\ln\left(\frac{ab}{c^2}\right) = \ln a + \ln b - \ln c^2$

$$= \ln a + \ln b - 2\ln c$$

5. $\ln\left(\frac{x+2}{x^2}\right) = \ln(x+2) - \ln x^2$

$$= \ln(x+2) - 2\ln x$$

6. $\log_2\left(\frac{2ac}{\sqrt{d}}\right)$

$$= \log_2 2 + \log_2 a + \log_2 c - \log_2 d^{\frac{1}{2}}$$

$$= 1 + \log_2 a + \log_2 c - \frac{1}{2}\log_2 d$$

7. $\log\left(\frac{x^3}{yz}\right) = \log x^3 - \log y - \log z$

$$= 3\log x - \log y - \log z$$

Solve each of the following equations for x by rewriting each side of the equation as a single logarithm of the same base. Then, set the arguments equal to one another. Then, in the boxes to the right, check the solution. If the equation has no solution, then specifically explain why.

$$\log x + 2\log 2 = \log(x+9)$$

$$\log x + \log 2^2 = \log(x+9)$$

$$\log(4x) = \log(x+9)$$

$$4x = x+9$$

$$3x = 9$$

$$x = 3$$

$$\log 3 + \log 4 = \log(3+9)$$

$$\log(12) = \log(12)$$

$$\log_3(x-4) - \log_3 2 = \log_3(x+1)$$

$$\log_3\left(\frac{x-4}{2}\right) = \log_3(x+1)$$

$$\frac{x-4}{2} = x+1$$

$$x-4 = 2x+2$$

$$-4 = x+2$$

$$-6 = x$$

No solution

$$\log_2(x+2) + \log_2 x = 3 \log_2 2$$

$$\log_2(x^2+2x) = \log_2(2^3)$$

$$x^2+2x = 8$$

$$x^2+2x-8=0$$

$$(x+4)(x-2)=0$$

~~$$x = -4, x = 2$$~~

$$x = 2$$

$$\log_2(2+2) + \log_2 2 = \log_2 2^3$$

$$\log_2 4 + 1 = 3$$

$$2 + 1 = 3$$

$$3 = 3$$

$$\ln(x+3) - \ln 2 = \ln(2x-1) + \ln 3$$

$$\ln\left(\frac{x+3}{2}\right) = \ln(6x-3)$$

$$\frac{x+3}{2} = 6x-3$$

$$x+3 = 12x-6$$

$$3 = 11x-6$$

$$9 = 11x$$

$$\frac{9}{11} = x$$

$$\ln(2x-3) = \ln x - 2 \ln 2$$

$$= \ln x - \ln 4$$

$$\ln(2x-3) = \ln\left(\frac{x}{4}\right)$$

$$2x-3 = \frac{x}{4}$$

$$8x-12 = x$$

$$7x = 12$$

$$x = \frac{12}{7}$$