Notes 8.1 Understanding the Value of a Logarithm

Logarithmic Form vs Exponential Form

$y = \log_b x$ iff $x = b^y$

Where $b > 0$ and $b \neq 1$

base argument

The expression $\log_b a$ is a logarithm and is read "log base b of a," where $a > 0$ and $b > 0$. Study the table of logarithmic equations below. In each box, write an exponential equation that is related with the given logarithmic equation.

$\log_3 27 = 3$ → Exponent	$\log_{10} 0.0001 = -4$ → Exponent	$\log_5 5 = 1$ → Exponent
$3^3 = 27$	$10^{-4} = 0.0001$	$5^1 = 5$

Make three conjectures about the value of a logarithm based on the numerical relationship between the value of the base (b) and the value of the argument (a) of the logarithm.

If $b = a$, then… $\log_b a = x$	The value of the logarithm = 1
If $b < a$, then… $\log_b a = x$	The value of the logarithm > 1
If $b > a$, then… $\log_b a = x$	The value of the logarithm < 1

A logarithm whose base is 10 is called a **<u>common logarithm</u>** and is written as $\log_{10} 100 = \log 100$. A logarithm with an unwritten base is understood to have a base of 10. Use a calculator to find the values of each of the following common logarithms.

$\log 5 = 0.699$	$\log 100 = 2$	$\log 115 = 2.061$

Based on your understanding of the value of a logarithm, what conclusion can you make about the values of the following logarithms?

Logarithmic Expression	**Find the value of the given logarithm. If the value is not an integer, between what two integer values do you think the value of the logarithmic expression lies?**	**Given the expression $\log_b a$, use the calculator to find the value of $\frac{\log a}{\log b}$.**
$\log_3 81 = x$ $3^x = 81$	$x = 4$	$\frac{\log 81}{\log 3} = 4$
$\log_2 \frac{1}{8} = x$ $2^x = \frac{1}{8}$	$x = -3$	$\frac{\log \frac{1}{8}}{\log 2} = -3$
$\log_2 9 = x$	$2^3 = 8$ and $2^4 = 16$ $3 < x < 4$	$\frac{\log 9}{\log 2} = 3.170$
$\log_3 36 = x$	$3^3 = 27$ and $3^4 = 81$ $3 < x < 4$	$\frac{\log 36}{\log 3} = 3.262$
$\log_4 21 = x$	$4^2 = 16$ and $4^3 = 64$ $2 < x < 3$	$\frac{\log 21}{\log 4} = 2.196$
$\log_3 105 = x$	$3^4 = 81$ and $3^5 = 243$ $4 < x < 5$	$\frac{\log 105}{\log 3} = 4.236$

What can you conclude about the values of $\log_b a$ and $\frac{\log a}{\log b}$?

Change of Base Formula: $\log_b a = \frac{\log a}{\log b}$

Developing the Natural Base, *e*

Consider the expression $\left(1+\frac{1}{n}\right)^n$. Complete the table below for the given values of *n*.

Table Calc

n	$\left(1+\frac{1}{n}\right)^n$
1	2
10	2.594
100	2.705
1000	2.717
10,000	2.718
100,000	2.718
1,000,000	2.718
10,000,000	2.718

$\pi \approx 3.14$

$e \approx 2.718$

As $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n =$ 2.718. This value is called *e* and is called the natural base.

A logarithm whose base is *e* is called a natural logarithm. $\log_e a$ is written as $\ln a$. All of the properties of logarithms that exist for bases other than *e* also exist for natural logarithms.

Logarithmic Expression	Given the expression $\log_b a$, use the calculator to find the value of $\frac{\log a}{\log b}$.	Given the expression $\log_b a$, use the calculator to find the value of $\frac{\ln a}{\ln b}$.
$\log_3 20$	$\frac{\log 20}{\log 3} = 2.727$	$\frac{\ln 20}{\ln 3} = 2.727$
$\log_2 \frac{1}{16}$	$\frac{\log \frac{1}{16}}{\log 2} = -4$	$\frac{\ln \frac{1}{16}}{\ln 2} = -4$
$\log_5 15$	$\frac{\log 15}{\log 5} = 1.683$	$\frac{\ln 15}{\ln 5} = 1.683$

What can you conclude about the values of $\log_b a$ and $\frac{\log a}{\log b}$ and $\frac{\ln a}{\ln b}$?

Change of Base Formula: $\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

Find the value of *x* in each of the logarithmic equations below by rewriting the equation as an exponential equation. Leave your answers in exact form, when possible. Otherwise, round your answers to three decimal places.

1. $\log_9 x = -\frac{2}{3}$

$9^{-2/3} = x$

$\left(\frac{1}{9}\right)^{2/3} = x$

$\sqrt[3]{\frac{1}{81}} = x$

2. $\ln(2x - 1) = 3$

$e^3 = 2x - 1$

$e^3 + 1 = 2x$

$\frac{1}{2}(e^3 + 1) = x$

3. $\ln(x + 3) = 5$

$e^5 = x + 3$

$e^5 - 3 = x$

4. $\log_3(2x + 1) = -3$

$3^{-3} = 2x + 1$

$\frac{1}{27} = 2x + 1$

$1 = 54x + 27$

$-26 = 54x$

$\frac{-26}{54} = x$

$\frac{-13}{27} = x$

5. $\log_{25}(5\sqrt{5}) = x$

$25^x = 5\sqrt{5}$

$(5^2)^x = 5^1 \cdot 5^{1/2}$

$5^{2x} = 5^{3/2}$

$2x = \frac{3}{2}$

$4x = 3$

$x = 3/4$

6. $\log_2 9 = x + 2$

$\frac{\log 9}{\log 2} = x + 2$

$\frac{\log 9}{\log 2} - 2 = x$