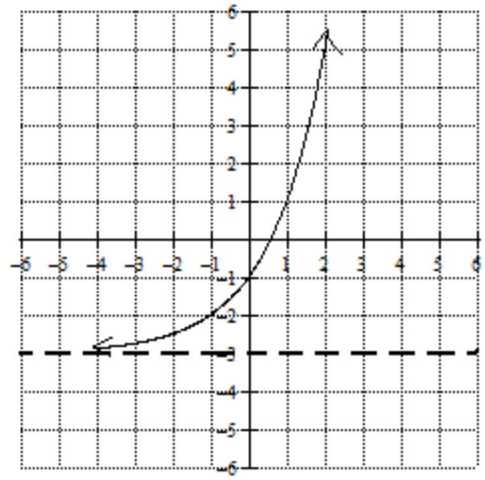
Notes 7.4 Finding Equations of Exponential Functions

The graph of an exponential function of the form $f(x) = a \cdot b^{x+1} + k$ is pictured to the right. Answer the following questions.



1. What is the value of k in the equation? Give a reason.

f(x) has a horizontal asymptote at $y = -3$

$\therefore k = -3$

2. Rewrite the equation using your value of k. Then, find use $f(-1) = -2$ to find the value of a.

$f(x) = a \cdot b^{x+1} + k$

$-2 = a \cdot b^{-1+1} - 3$

$1 = a \cdot b^0$

$1 = a \cdot 1$

$1 = a$

3. Use another point on the graph to determine the value of b. What is the equation of $f(x)$?

$\boxed{(0,-1)}$

$f(x) = 1 \cdot b^{x+1} - 3$

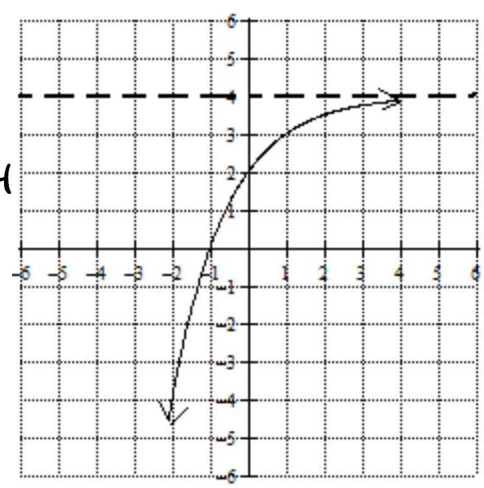
$-1 = b^{0+1} - 3$

$2 = b^1$

$2 = b$

$f(x) = 2^{x+1} - 3$

The graph of an exponential function $g(x) = a \cdot b^x + k$. Determine the values of a, b, and k.



$\boxed{(0,2)}$

$g(x) = a \cdot b^x + 4$

$2 = a \cdot b^0 + 4$

$-2 = a \cdot 1$

$-2 = a$

$\boxed{(1,3)}$

$g(x) = -2 \cdot b^x + 4$

$3 = -2 \cdot b^1 + 4$

$-1 = -2 \cdot b^1$

$\frac{1}{2} = b$

$a = -2,\ b = \frac{1}{2},\ k = 4$

Do the values of a and c that you found for $g(x)$ make sense in terms of the graph being an exponential growth function? Explain your reasoning.

$g(x) = -2\left(\frac{1}{2}\right)^x + 4$

$g(x) = -2(2)^{-x} + 4$

$a < 0$ and $c < 0$.

$\therefore$ g(x) has 2 reflections

$\therefore$ g(x) has exponential growth

Find the equation of the exponential functions described below. Show your work.

Find the equation $f(x) = a \cdot b^x$ if $f(0) = 3$ and $f(1) = 15$.

$f(x) = a \cdot b^x$

$3 = a \cdot b^0$

$3 = a \cdot 1$

$3 = a$

$f(x) = 3 \cdot b^x$

$15 = 3 \cdot b^1$

$5 = b$

$f(x) = 3(5)^x$

Find the equation of $g(x) = a \cdot b^x + k$ if $g(0) = 2$ and $g(-4) = -2$. Additionally, $\lim_{x\to\infty} g(x) = 3$. HA $y = 3$

$g(x) = a \cdot b^x + 3$

$2 = a \cdot b^0 + 3$

$-1 = a \cdot 1$

$-1 = a$

$g(x) = -1 \cdot b^x + 3$

$-2 = -1 \cdot b^{-4} + 3$

$-5 = -1 \cdot b^{-4}$

$5 = b^{-4}$

$5 = \frac{1}{b^4}$

$5b^4 = 1$

$b^4 = \frac{1}{5}$

$b = \left(\frac{1}{5}\right)^{\frac{1}{4}}$

$g(x) = -1\left(\left(\frac{1}{5}\right)^{\frac{1}{4}}\right)^x + 3$

$= -1(5)^{-\frac{1}{4}x} + 3$

A graphing calculator can be used to perform an exponential regression analysis that will find an exponential function in the form $f(x) = a \cdot b^x$. Test the capability on the function $f(x)$ above. You will perform this just as you performed linear and quadratic regressions in algebra 2. Write your procedure in the space below.

1. $\boxed{\text{STAT}}$ choose $\boxed{\text{EDIT}}$ → enter x-values into L_1
   → enter y-values into L_2

2. $\boxed{\text{STAT}}$ choose $\boxed{\text{CALC}}$ → choose $\boxed{\text{ExpReg}}$

A pharmacologist is testing a new antibiotic. At the beginning of the first day of an experiment, there were 2500 bacteria in a culture dish. At the end of each day, the number of bacteria present in the dish was recorded in the table below.

(0, 2500)

Day #	# of Bacteria Present
1	2157
2	1802
3	1698
4	1429
5	1130
6	800

a. Based on the data in the table, is this an example of an exponential growth or decay? Explain your answer.

The number of bacteria decrease

∴ Exponential decay.

b. Using the regression capabilities of the graphing calculator, find the function, $f(x) = a \cdot b^x$, that models the data in the table.

$f(x) = 2629.226(0.838)^x$

c. How many bacteria will be present at the end of 10 days? Show your work.

$f(10) = 2629.226(0.838)^{10}$

$f(10) = 449.023$

After 10 days, about 449 bacteria remain in the culture.

d. On what day of the experiment would the pharmacologist expect the number of bacteria in the culture dish to be below 100? Explain your reasoning.

$100 = 2629.226(0.838)^x$

$\boxed{\begin{array}{l} Y_1 = 100 \\ Y_2 = 2629.226 \end{array}} \Rightarrow x \approx 18.498$

On the 19th day, the number of bacteria will be 100, so beyond that point the population will be below 100.

e. Will the bacteria ever be completely gone from the culture dish? Explain why or why not using your knowledge of the graphs of exponential functions.

f(x)'s graph has a horizontal asymptote at y=0, mean f(x), the number of bacteria, will never reach zero and be completely gone.