Notes 7.3 Classifying Exponential Functions as Growth or Decay
Graphical, Numerical and Analytical Approaches

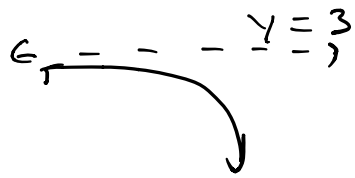
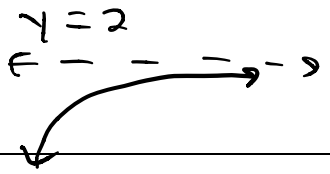
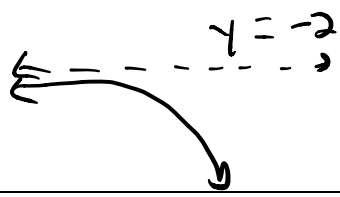
You should now be able to make three preliminary inferences as to how to determine, based solely on the equation, if an exponential function will have a graph that represents a growth or a decay. Make those inferences in the spaces below.

1. If the graph has no reflections, it has growth

2. If the graph has 2 reflections, it has growth.

3. If the graph has 1 reflection, it has decay

What is the only scenario not included in the above inferences?

Function **Is the value of $b > 1$ or is $0 < b < 1$? If $0 < b < 1$, reciprocate b.**	**Number of reflections indicated in the equation**	**Sketch of the graph**	**Is the graph a growth or a decay?**
1. $f(x) = -\left(\frac{2}{3}\right)^{-x+2} + 3$ $f(x) = -\left(\frac{3}{2}\right)^{x-2} + 3$	1		Decay
2. $g(x) = -2^{-x-1} + 2$ $g(x) = -(2)^{-(x+1)} + 2$	2		Growth
3. $h(x) = -\left(\frac{1}{2}\right)^{-x+3} - 2$ $h(x) = -(2)^{x-3} - 2$	1		Decay

Determine if the following functions are examples of exponential growth or decay functions. State the reasoning that leads to your conclusions. Also, state the equation of the horizontal asymptote for each function.

1) $F(x) = -(2)^x + 3$

$a < 0$ ∴ F(x) has a vertical reflection
$c > 0$ ∴ F(x) does Not have a horizontal reflection.
∴ F(x) has one reflection
∴ F(x) has exponential DECAY.

F(x) has a constant of 3
∴ the HA is at $y = 3$.

2) $G(x) = (0.25)^{-x} - 2$

$G(x) = (4)^x - 2$
$a > 0$ ∴ G(x) does Not have a vertical reflection
$c > 0$ ∴ G(x) does Not have a horizontal reflection.
∴ G(x) has NO reflections.
∴ G(x) has exponential growth.

$= -2\left(\frac{3}{5}\right)^{-(x-1)} + 3$

G(x) has a constant of -2
∴ the HA is at $y = -2$.

3) $H(x) = -2(3/2)^{-x+1} + 3$

$a < 0$ ∴ H(x) has a vertical reflection
$c > 0$ ∴ H(x) has a horizontal reflection.
∴ H(x) has two reflections
∴ H(x) has exponential growth.

H(x) has a constant of 3
∴ the HA is at $y = 3$.

Identify each indicated property below. For a, c, and k, write the most specific conclusion that you can make about those values from the graph.

Growth or (Decay) Justification: The graph is decreasing at an increasing rate

Left End Behavior $\lim_{x \to -\infty} f(x) = 2$

Right End Behavior $\lim_{x \to \infty} f(x) = -\infty$

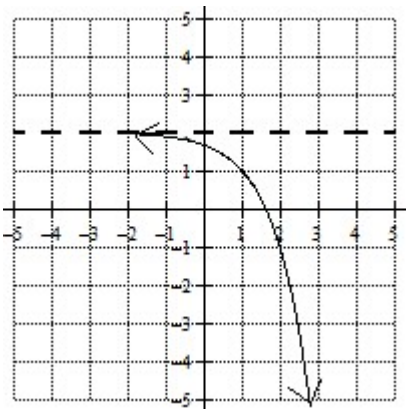
Equation of horizontal asymptote $y = 2$

a: < 0 b/c $f(x)$ is below HA

c: > 0 b/c $\lim_{x \to \infty} f(x) = -\infty$ ($f(x)$ goes away from HA)

k: $= 2$ b/c HA is $y = 2$

$f(x) = a \cdot b^{c(x-h)} + k$



Growth or (Decay) Justification: The graph is decreasing at a decreasing rate.

Left End Behavior $\lim_{x \to -\infty} f(x) = \infty$

Right End Behavior $\lim_{x \to \infty} f(x) = -1$

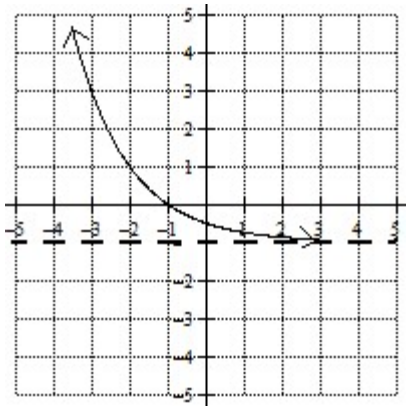
Equation of horizontal asymptote $y = -1$

a: > 0 b/c $f(x)$ is above HA

c: < 0 b/c $\lim_{x \to \infty} f(x) = -1$ ($f(x)$ goes away from HA)

k: $= -1$ b/c HA is $y = -1$

$f(x) = a \cdot b^{c(x-h)} + k$



(Growth) or Decay Justification: The graph is increasing at an increasing rate

Left End Behavior $\lim_{x \to -\infty} f(x) = -2$

Right End Behavior $\lim_{x \to \infty} f(x) = \infty$

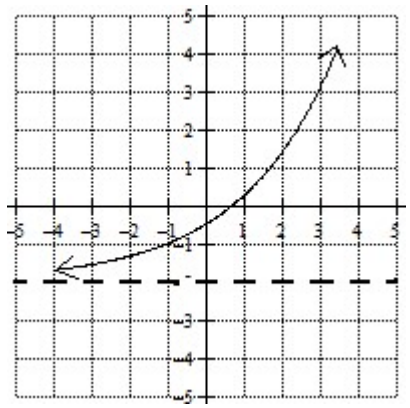
Equation of horizontal asymptote $y = -2$

a: > 0 b/c $f(x)$ is above HA

c: > 0 b/c $\lim_{x \to \infty} f(x) = \infty$ ($f(x)$ goes away from HA)

k: $= -2$ b/c HA is $y = -2$

$f(x) = a \cdot b^{c(x-h)} + k$



Let's consider a numerical example of the exponential function, $H(x) = a \cdot b^{c(x-h)} + k$

x	−9	−5	−3	−1	1	3	9
$H(x)$	513	33	9	3	1.5	1.125	1.002

Is $H(x)$ a growth or decay? Give a reason.

H(x) is decreasing at a decreasing rate.

∴ H(x) is exponential decay

State the left end behavior.

$\lim_{x \to -\infty} H(x) = \infty$

State the right end behavior.

$\lim_{x \to \infty} H(x) = 1$

Identify the equation of the horizontal asymptote. Give a reason. What is the value of k?

∴ y = 1

∴ K = 1

Is the value of $a < 0$ or is $a > 0$. Give a reason

All the y-values of H(x) are above the HA of y = 1.

∴ a > 0

What is the value of c in the equation of $H(x)$? Give a reason.

H(x) is growing toward the HA of y = 1

∴ c < 0

Identify the domain and range of $H(x)$.

Domain $(-\infty, \infty)$

Range $(1, \infty)$