# Notes 7.1 Properties of Exponents and Solving Exponential Equations

### *Analytical, Numerical, and Graphical Connections*

**Property of Exponents**

Product Property $a^m \cdot a^n = a^{m+n}$

Quotient Property $\frac{a^m}{a^n} = a^{m-n}$

Power of a Power Property $(a^m)^n = a^{m \cdot n}$

Negative Exponent Property $(a)^{-n} = \left(\frac{1}{a}\right)^n$

Zero Exponent Property $a^0 = 1$

Rational Exponent Definition $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Property of Equality If $a^x = a^y$, then $x = y$

**Rewrite each expression below as a single base.**

$5^{2x-3} \cdot 5^{5-4x} = 5^{(2x-3)+(5-4x)}$
$= 5^{-2x+2}$

$\frac{2^x \cdot 2^{3x+4}}{2^{2x+1} \cdot 2^{x-5}} = \frac{2^{(x)+(3x+4)}}{2^{(2x+1)+(x-5)}} = \frac{2^{4x+4}}{2^{3x-4}} = 2^{(4x+4)-(3x-4)}$
$= 2^{x+8}$

$(2^3)^{x-2} \cdot 2^{2-2x} = 2^{3(x-2)} \cdot 2^{2-2x}$
$= 2^{3x-6} \cdot 2^{2-2x}$
$= 2^{(3x-6)+(2-2x)}$
$= 2^{x-4}$

$3^{-2x+2} = 3^{-1\cdot(2x-2)}$
$= \left(\frac{1}{3}\right)^{2x-2}$

$\frac{4^{2x+2} \cdot 2^{x-1}}{2^{5x+3}} = \frac{(2^2)^{2x+2} \cdot 2^{x-1}}{2^{5x+3}} = \frac{2^{4x+4} \cdot 2^{x-1}}{2^{5x+3}}$
$= \frac{2^{5x+3}}{2^{5x+3}} = 2^0 = 1$

$\sqrt{3^{2x+2} \cdot 9^{x-4}} = \left(3^{2x+2} \cdot (3^2)^{x-4}\right)^{1/2}$
$= \left(3^{2x+2} \cdot 3^{2x-8}\right)^{1/2}$
$= \left(3^{4x-6}\right)^{1/2}$
$= 3^{2x-3}$

$3^{2x+4} = 3^{x-8}$
$2x+4 = x-8$
$x+4 = 8$
$x = 12$

Simplify each expression below using the properties of exponents.

1. $(3^1x^3y^5)^3$

$$= 3^3x^9y^{15}$$
$$= 27x^9y^{15}$$

2. $(3ab^2)\cdot(2a^2b^3)^3$

$$= 3ab^2\cdot 2^3a^6b^9$$
$$= 3\cdot 8\cdot a^7b^{11}$$
$$= 24a^7b^{11}$$

3. $\frac{12a^3}{15a^5}$

$$= \frac{4}{5a^2}$$

4. $\left(\frac{3a^2b^3}{5ab^2}\right)^2$

$$= \frac{3^2a^4b^6}{5^2a^2b^4}$$
$$= \frac{9a^2b^2}{25}$$

5. $\left(\frac{x^2y^{-3}}{x^5y^2}\right)^{-3}$

$$= \frac{x^{-6}y^9}{x^{-15}y^{-6}} = x^9y^{15}$$

6. $\left(\frac{8x^3}{27y^6}\right)^{\frac{2}{3}}$

$$= \left(\frac{8^{1/3}x^{3/3}}{27^{1/3}y^{6/3}}\right)^2$$
$$= \left(\frac{2x}{3y^2}\right)^2$$
$$= \frac{4x^2}{9y^4}$$

7. $\left(\frac{25}{16a^4}\right)^{-\frac{3}{2}}$

$$= \left(\frac{16a^4}{25}\right)^{3/2}$$
$$= \left(\frac{16^{1/2}a^{4/2}}{25^{1/2}}\right)^3$$
$$= \left(\frac{4a^2}{5}\right)^3$$
$$= \frac{64a^6}{125}$$

8. $\left(\frac{8a^9}{125b^3}\right)^{-\frac{2}{3}}$

$$= \left(\frac{125b^3}{8a^9}\right)^{2/3}$$
$$= \left(\frac{125^{1/3}b^{3/3}}{8^{1/3}a^{9/3}}\right)^2$$
$$= \left(\frac{5b}{2a^3}\right)^2$$
$$= \frac{25b^2}{4a^6}$$

Rewrite each of the following expressions as a single exponential expression in the form $a^m$.

9. $25^{3x+2}\cdot(5^{x-2})^3$

$$= 5^{2(3x+2)}\cdot 5^{3x-6}$$
$$= 5^{6x+4}\cdot 5^{3x-6}$$
$$= 5^{9x-2}$$

10. $\sqrt[3]{\frac{8^{2x-1}}{2^{3x-6}}}$

$$= \left(\frac{2^{3(2x-1)}}{2^{3x-6}}\right)^{1/3}$$
$$= \left(\frac{2^{6x-3}}{2^{3x-6}}\right)^{1/3}$$
$$= \left(2^{3x+3}\right)^{1/3}$$
$$= 2^{x+1}$$

Rewrite each side of the following equations as a single exponential expression of the same base. Then, solve the equation by setting the exponents equal to each other.

11. $27^{x-2} = 9^{2x+5}$

$3^{3(x-2)} = 3^{2(2x+5)}$

$3^{3x-6} = 3^{4x+10}$

$3x-6 = 4x+10$

$-6 = x+10$

$-16 = x$

12. $\left(\frac{1}{4}\right)^{x+2} = 8^{x-4}$

$\left(2^{-2}\right)^{x+2} = 2^{3(x-4)}$

$2^{-2x-4} = 2^{3x-12}$

$-2x-4 = 3x-12$

$5x-4 = -12$

$5x = 8$

$x = 8/5$

13. $\sqrt{4^{2x-7}} = (2^{x-1})^3$

$\left(2^{2(2x-7)}\right)^{1/2} = 2^{3x-3}$

$2^{2x-7} = 2^{3x-3}$

$2x-7 = 3x-3$

$-7 = x-3$

$-4 = x$

14. $\frac{16^{x+2}}{8^{x-1}} = 4^{3x+1}$

$\frac{2^{4(x+2)}}{2^{3(x-1)}} = 2^{2(3x+1)}$

$\frac{2^{4x+8}}{2^{3x-3}} = 2^{6x+2}$

$2^{x+11} = 2^{6x+2}$

$x+11 = 6x+2$

$11 = 5x+2$

$9 = 5x$

$9/5 = x$

**The Graphical Connection of Solving Exponential Equations**

Consider the equation $2^{x+3} = \left(\frac{1}{2}\right)^{x+1}$. Solve this equation by rewriting each side of the equation with the same base and then setting the exponents equal to each other.

$2^{x+3} = \left(2^{-1}\right)^{x+1}$

$2^{x+3} = 2^{-x-1}$

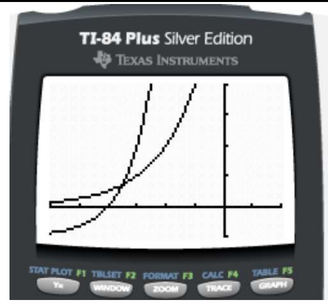
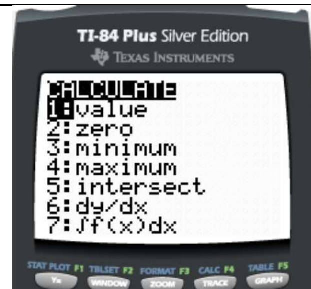
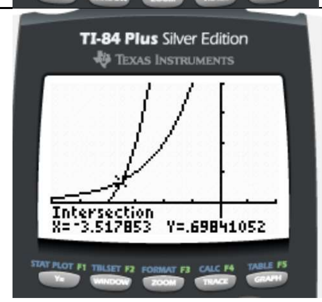
$x+3 = -x-1$

$2x+3 = -1$

$2x = -4$

$x = -2$

To solve exponential equations, both sides of the equation must be rewritten so that the bases are the same. However, sometimes this is not possible. For example, in the equation $2^{x+3} = 3^{x+4} - 1$. The bases are 2 and 3, neither of which can be rewritten as a power of the other. Therefore, at this point, the only way that we can solve this equation is to graph each side of the equation in the calculator and use the intersect function to find the point of intersection, which we showed previously is the solution to the equation.

| | |
|---|---|
| 1. Enter $2^{x+3}$ into the $Y_1$ in the calculator | $Y_1$: 2^(x + 3) |
| 2. Enter $3^{x+4} - 1$ into $Y_2$ in the calculator | $Y_2$: 3^(x+4) – 1 |
| 3. Hit the GRAPH button so that both functions are now displayed. You must be able to CLEARLY see the point of intersection so there may need to be some adjustment made to the WINDOW that is being viewed. You'll see, I changed my window to be XMIN: –6<br>XMAX: 2<br>YMIN: –1<br>YMAX: 4 |  |
| 4. With the graphs displayed on the screen, go to the CALC menu by hitting the 2ND and TRACE keys. Choose option #5 intersect. |  |
| 5. You will now be taken back to the home screen and "Mr. Blinky" appears. He is asking, "Am I on the first curve?" Hit ENTER to tell him "Yes." He then jumps to the second curve and asks, "Am I on the second curve?" Hit ENTER to tell him "Yes." Mr. Blinky then asks, "Do you now want me to guess what the intersection is?" Again, you hit ENTER to say, "Heck yeah, I want you to guess!!" |  |
| 6. According to the calculator, then the solution to the equation is $x = –3.518$. | IMPORTANT NOTE: Sometimes, there are two points of intersection of the graphs. You will have to find these one at a time. Mr. Blinky will find the point of intersection closest to where he is located. |

Use this method to find the solution of each of the equations below.

15. $2^{x+1} = 3^{-x+2}$

$x \approx 0.839$

16. $2^{x-3} - 2 = -2^{x-2} + 2$

$x \approx 3.415$

17. $\left(\frac{1}{2}\right)^{x-2} + 2 = -3^{x-4} + 4$

$x \approx 1.078$

$x \approx 4.550$