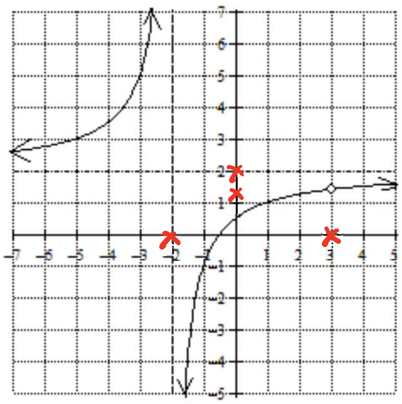
Notes 5.6 Discontinuities in Rational Functions

Graphical and Numerical Approaches

1. $f(x) = \frac{2x^2-5x-3}{x^2-x-6}$

hole $\left(3, \frac{7}{5}\right)$



$\lim_{x\to-\infty} f(x) =$ 2

$\lim_{x\to\infty} f(x) =$ 2

$\lim_{x\to 3^-} f(x) =$ 7/5

$\lim_{x\to 3^+} f(x) =$ 7/5

$\lim_{x\to -2^-} f(x) =$ ∞

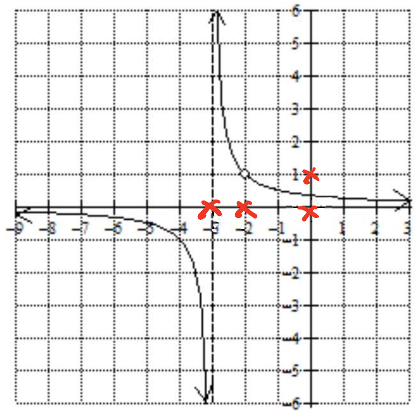
$\lim_{x\to -2^+} f(x) =$ $-\infty$

Domain: $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

Range: $(-\infty,\frac{7}{5})\cup(\frac{7}{5},2)\cup(2,\infty)$

3. $h(x) = \frac{x+2}{x^2+5x+6}$

hole $(-2,1)$



$\lim_{x\to-\infty} h(x) =$ 0

$\lim_{x\to\infty} h(x) =$ 0

$\lim_{x\to -2^-} h(x) =$ 1

$\lim_{x\to -2^+} h(x) =$ 1

$\lim_{x\to -3^-} h(x) =$ $-\infty$

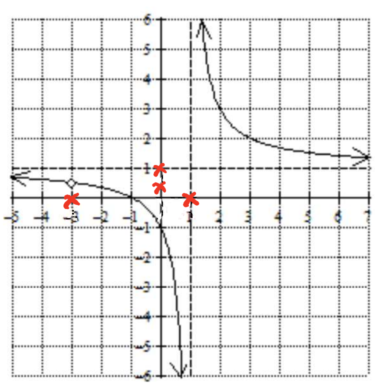
$\lim_{x\to -3^+} h(x) =$ ∞

Domain: $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

Range: $(-\infty,0)\cup(0,1)\cup(1,\infty)$

2. $g(x) = \frac{x^2+4x+3}{x^2+2x-3}$

hole $\left(-3, \frac{1}{2}\right)$



$\lim_{x\to-\infty} g(x) =$ 1

$\lim_{x\to\infty} g(x) =$ 1

$\lim_{x\to -3^-} g(x) =$ $\frac{1}{2}$

$\lim_{x\to -3^+} g(x) =$ $\frac{1}{2}$

$\lim_{x\to 1^-} g(x) =$ $-\infty$

$\lim_{x\to 1^+} g(x) =$ ∞

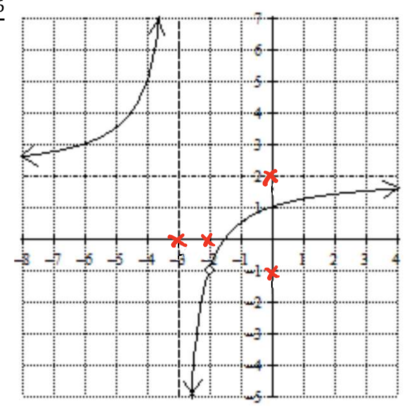
Identify the domain and range of $g(x)$:

Domain: $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$

Range: $(-\infty,\frac{1}{2})\cup(\frac{1}{2},1)\cup(1,\infty)$

4. $p(x) = \frac{2x^2+7x+6}{x^2+5x+6}$

hole $(-2,-1)$



$\lim_{x\to-\infty} p(x) =$ 2

$\lim_{x\to\infty} p(x) =$ 2

$\lim_{x\to -2^-} p(x) =$ -1

$\lim_{x\to -2^+} p(x) =$ -1

$\lim_{x\to -3^-} p(x) =$ ∞

$\lim_{x\to -3^+} p(x) =$ $-\infty$

Domain: $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$

Range: $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

Based on your observations from the graphical analysis of the four functions on the previous page, fill in the blanks in each statement below or answer the question asked.

1. In the graph of $f(x)$ if $\lim_{x\to\pm\infty} f(x) = a$, then the graph of $f(x)$ has a

horizontal asymptote at $y=a$

2. In the graph of $f(x)$ if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = b$, but $f(a) \neq b$, then the graph of $f(x)$ has a

point discontinuity (hole) at (a,b)

3. In the graph of $f(x)$ if $\lim_{x\to a^\pm} f(x) = \pm\infty$, then the graph of $f(x)$ has a

vertical asymptote at $x=a$

4. How do you determine the domain of the rational functions you have seen so far?

Domain = $(-\infty,\infty)$ excluding x-values of vertical asymptotes and point discontinuities.

5. How do you determine the range of the rational functions you have seen so far?

Range = $(-\infty,\infty)$ excluding y-values of horizontal asymptotes and point discontinuities.

The table below shows selected values on the graph of a rational function, $F(x)$. Use the information in the table to answer the questions below.

x	−500	−0.501	−0.5	−0.499	1.9	2	2.1	500
$F(x)$	1.499	−248.5	Undefined	251.5	1.604	Undefined	1.596	1.501

VA @ $x = -1/2$ Hole (2, 1.6)

a. Identify the equation of any horizontal asymptote of the graph of $F(x)$. Give a reason for your answer.

$\lim_{x\to-\infty} F(x) = 1.5$, $\lim_{x\to\infty} F(x) = 1.5$ ∴ $F(x)$ has a horizontal asymptote at $y = 1.5$

b. Identify the equation of any vertical asymptote of the graph of $F(x)$. Give a reason for your answer.

$\lim_{x\to-\frac{1}{2}^-} F(x) = -\infty$, $\lim_{x\to-\frac{1}{2}^+} F(x) = \infty$ $F(x)$ has a vertical asymptote at $x = -\frac{1}{2}$

c. Identify the coordinates of any hole in the graph of $F(x)$. Give a reason for your answer.

$\lim_{x\to2^-} F(x) = 1.6$, $\lim_{x\to2^+} F(x) = 1.6$, * $F(2) \neq 1.6$ ∴ $F(x)$ has a hole at (2, 1.6)

d. State the domain and range of the graph of $F(x)$.

D: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 2) \cup (2, \infty)$

R: $(-\infty, 1.5) \cup (1.5, 1.6) \cup (1.6, \infty)$

The table below shows function values for a rational function, $G(x)$. The equation of $G(x)$ is such that the denominator of the function is the quadratic expression $2x^2 - 4x - 6$. $= 2(x^2 - 2x - 3) = 2(x-3)(x+1)$

x	-500	-1.0001	-1	-.999	2	2.9999	3	3.00001	500
$G(x)$	1.9801	-0.4999	undefined	-0.501	-8	-99,998	undefined	1,000,002	2.020

Hole $(-1, -\frac{1}{2})$ VA @ $x = 3$

a. What is the domain of $G(x)$?

$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

b. Does either factor in the denominator also exist in the numerator? Give a reason for your answer.

$\lim_{x\to-1^-} G(x) = \lim_{x\to-1^+} G(x) = -\frac{1}{2}$ & $G(-1) \neq -\frac{1}{2}$ *

∴ $G(x)$ has Point Discontinuity at $x = -1$

∴ The factor of $(x+1)$ exists in both numerator & denominator.

c. Does either factor of the denominator not exist in the numerator? Give a reason for your answer.

$\lim_{x\to3^-} G(x) = -\infty$ and $\lim_{x\to3^+} G(x) = +\infty$

∴ $G(x)$ has a VA at $x = 3$

∴ $(x-3)$ is a factor in the denominator but not the numerator.

d. Based on the end behavior, where does $G(x)$ have a horizontal asymptote? Give a reason for your answer.

$\lim_{x\to-\infty} G(x) = 2$ & $\lim_{x\to\infty} G(x) = 2$

∴ $G(x)$ has a HA at $y = 2$.

e. What must the leading coefficient of the numerator of the equation of $G(x)$ have to be? Explain your reasoning.

The ratio of the lead coefficients of the numerator and denominator, $\frac{a}{2}$, must simplify to 2 b/c $G(x)$ has a HA at $y = 2$.

∴ "a", the leading coefficient of the numerator is 4.