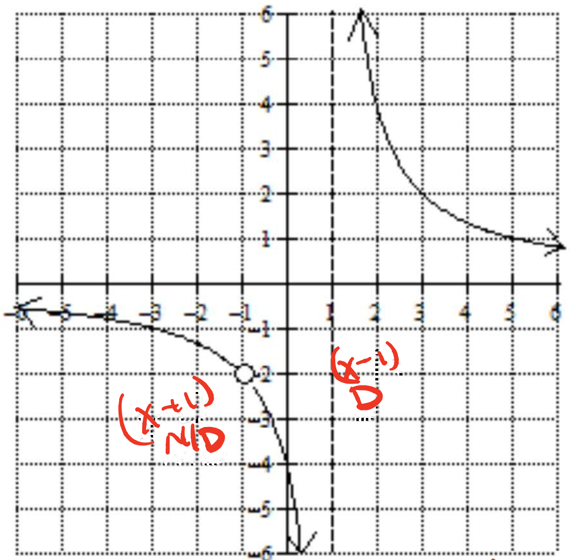
Notes 5.5 Graphing Rational Functions

Determine a possible equation that would represent each of the rational functions graphed below. Leave your equations in both factored and standard form.



Hole $x = -1$

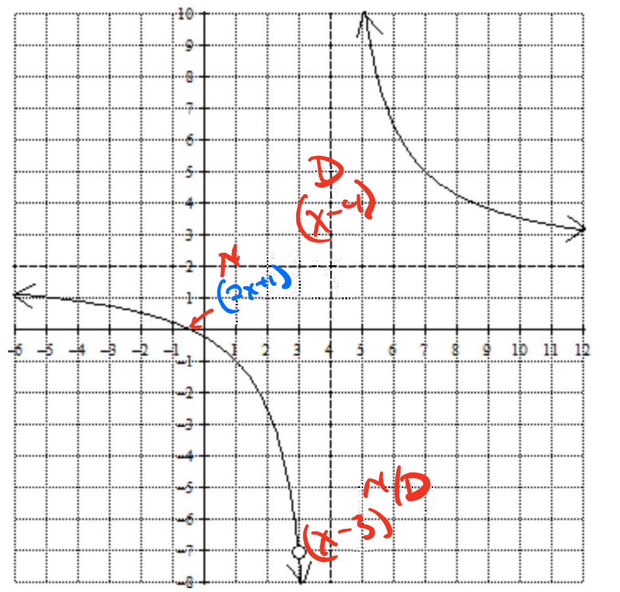
$(x+1) = 0$

$$f(x) = \frac{(x+1)}{(x-1)(x+1)}$$

VA: $x = 1$

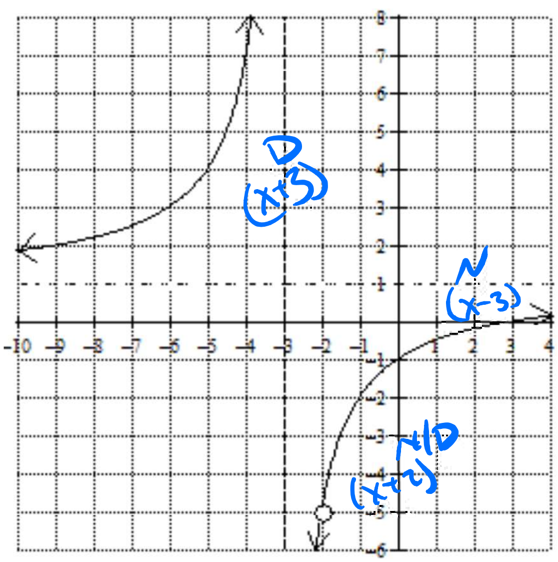
$(x-1) = 0$

$$= \frac{4x+4}{x^2-1}$$



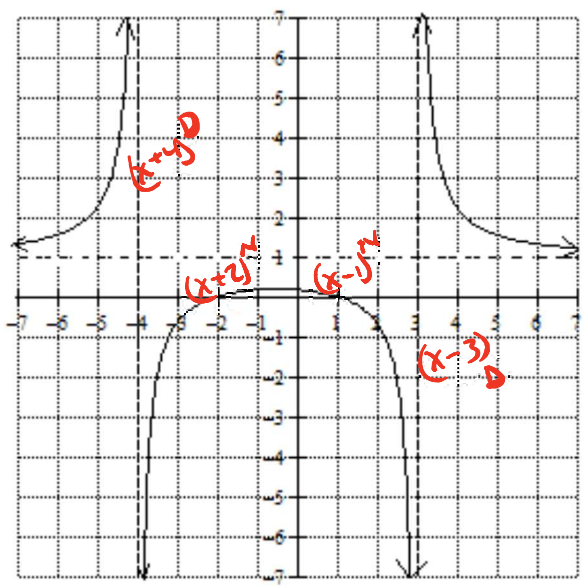
$$g(x) = \frac{(2x+1)(x-3)}{(x-4)(x-3)} = \frac{2x^2-5x-3}{x^2-7x+12}$$

$= \quad x - 3$



$$h(x) = \frac{(x+2)(x-3)}{(x+2)(x+3)}$$

$$= \frac{x^2 - x - 6}{x^2+5x+6}$$



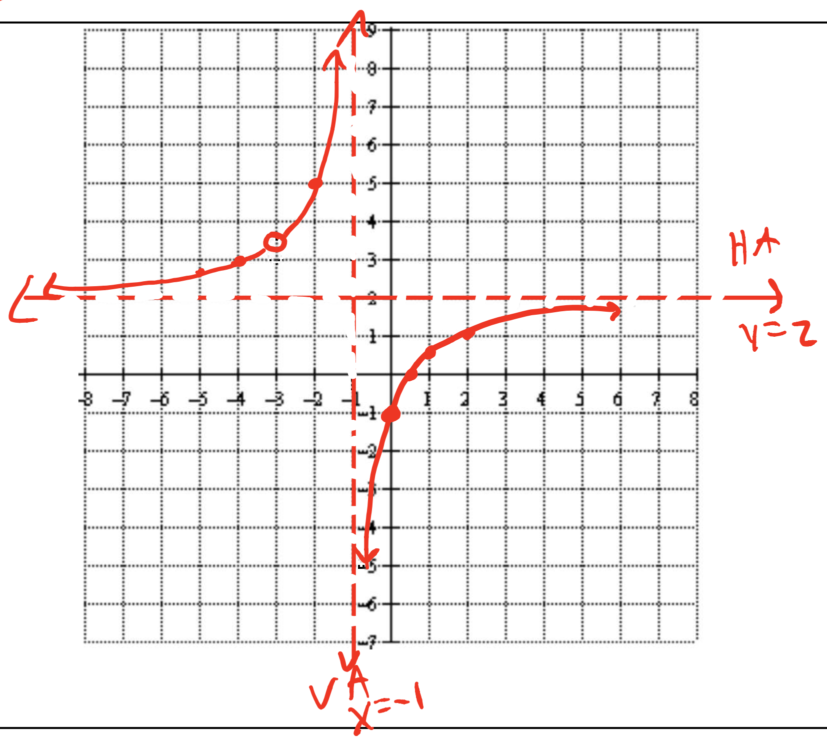
$$k(x) = \frac{(x+2)(x-1)}{(x+4)(x-3)}$$

$$k(x) = \frac{x^2+x-2}{x^2+x-12}$$

Given the equations of the rational functions below, develop a graph of each function.

HA $y=\frac{2}{1}$ y-int $=-\frac{3}{3}$

$$f(x)=\frac{2x^2+5x-3}{x^2+4x+3}$$

What is the equation of the function written in completely factored form?	zero @ $x=\frac{1}{2}$ $f(x)=\frac{(2x-1)(\cancel{x+3})}{(x+1)(\cancel{x+3})}$ Hole @ $x=-3$ VA @ $x=-1$
If any exist, identify the vertical asymptotes? Explain how you know that they are vertical asymptotes.	f(x) has a VA @ $x=-1$ b/c $(x+1)$ is a noncanceling factor in the denominator
Does the function have any holes in the graph? Explain why or why not. What are the coordinates where the hole(s) exist(s)?	f(x) has a hole @ $x=-3$ b/c $(x+3)$ is a canceling factor of numerator & denominator Hole $(-3, 7/2)$ $f(-3)=\frac{2(-3)-1}{(-3)+1}=\frac{-6-1}{-2}=\frac{-7}{-2}=7/2$
If any exist, identify the horizontal asymptotes. Explain how you know that they are horizontal asymptotes.	f(x) has a horizontal asymptote at $y=\frac{2}{1}=2$ b/c the numerator and denominator are the same degree.
What is/are the zero(s) of the function? Show your work.	f(x) has a zero at $x=1/2$ because $(2x-1)$ is a noncancelling factor of the numerator.
What are the domain and range of the function? Give your answer in interval notation.	D: $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$ R: $(-\infty,2)\cup(2,\frac{7}{2})\cup(\frac{7}{2},\infty)$
Sketch a detailed graph of the function on the grid to the right. You will need to use a minimum of 8 points—4 points on each branch. y-int $=\frac{-3}{3}=-1$ x | $\frac{2x-1}{x+1}$ 1 | $\frac{1}{2}$ 2 | $3/3=1$ -2 | $-5/-1=5$ -4 | $-9/-3=3$ -5 | $\frac{-11}{-4}\approx 2.75$	

SA y-int = $\frac{2}{8}$

$$g(x) = \frac{x + 2}{x^2 + 6x + 8}$$

y-int = $\frac{2}{8} = \frac{1}{4}$

What is the equation of the function written in completely factored form?	$g(x) = \frac{\cancel{x+2}}{\cancel{(x+2)}(x+4)}$ hole @ $x=-2$ VA @ $x=-4$ no zero
If any exist, identify the vertical asymptotes? Explain how you know that they are vertical asymptotes.	$g(x)$ has a VA @ $x=-4$ b/c $(x+4)$ is a noncanceling factor of the denominator.
Does the function have any holes in the graph? Explain why or why not. What are the coordinates where the hole(s) exist(s)?	$g(x)$ has a hole @ $x=-2$ b/c $(x+2)$ is a canceling factor of the numerator & denominator. Hole $(-2, \frac{1}{2})$ — $g(-2) = \frac{1}{-2+4} = \frac{1}{2}$
If any exist, identify the horizontal asymptotes. Explain how you know that they are horizontal asymptotes.	$g(x)$ has a HA @ $y=0$ b/c the degree of the numerator is less than the degree of the denominator.
What is/are the zero(s) of the function? Show your work.	$g(x)$ has no zeros b/c there is no noncanceling factor in the numerator.
What are the domain and range of the function? Give your answer in interval notation.	D: $(-\infty,-4)\cup(-4,-2)\cup(-2,\infty)$ R: $(-\infty,0)\cup(0,\frac{1}{2})\cup(\frac{1}{2},\infty)$

Sketch a detailed graph of the function on the grid to the right. You will need to use a minimum of 8 points—4 points on each branch.

y-int = $\frac{2}{8} = \frac{1}{4}$

x	$\frac{1}{x+4}$
-5	$\frac{1}{-1} = -1$
-6	$\frac{1}{-2}$
-7	$\frac{1}{-3}$
-8	$\frac{1}{-4}$
-3	$\frac{1}{1} = 1$
-1	$\frac{1}{3}$