Notes 5.4 Connecting the Equation of a Rational Function to Its Graph

A Focus on Asymptotic Behavior

Quick review of connections between factors and terms in the equation and what their graphical consequences are.

$$g(x) = \frac{-2x^2 + x + 3}{x^2 + 3x + 2} = \frac{(-2x+3)(x+1)}{(x+2)(x+1)}$$

y-int zero VA Hole

Non-cancelling Factors in the Denominator	Non-cancelling Factors in the Numerator	Cancelling Factors in the Numerator and Denominator	The Quotient of the Constant Terms
When the equation has a factor in the denominator that does not cancel out, then the graph has a(n) <u>vertical Asymptote</u> at the x value that makes that factor equal 0.	When the equation has a factor in the numerator that does not cancel out, then the graph has a(n) <u>zero</u> at the x value that makes that factor equal 0.	When the equation has a factor in the numerator and denominator that does cancel out, then the graph has a(n) <u>Hole</u> at the x value that makes that factor equal 0.	When the equation is in standard form, the <u>y-int</u> is the point $(0, c)$ where c is the quotient of the constant terms of the numerator and denominator.

Existence of Horizontal Asymptotes

Three rules that govern the existence and determination of the horizontal asymptote of a rational function. Assume "n" is the degree of the numerator and "d" is the degree of the denominator.

1. If n < d, then HA @ $y=0$
2. If n = d, then HA @ $y = \frac{\text{lead coefficient}}{\text{lead coefficient}}$
3. If n > d, then NO HA

Existence of Slant Asymptotes

This will always occur when the degree of the numerator is exactly 1 greater than the degree of the denominator. Slant asymptotes are oblique lines. Oblique lines are defined to be lines that are not vertical or horizontal. They have a slope and a y–intercept.

$n = d + 1$

To find the slant asymptote,

Divide the numerator by denominator using long division. Looking linear EQ (IGNORE Remainder)

Determine if each equation has a horizontal or slant asymptote and give a reason. Then find the horizontal or slant asymptote.

1. $f(x) = \frac{x+3}{x^2-x-6}$ HA $y=0$

f(x) has a horizontal asymptote at $y=0$ b/c the degree of numerator < degree of the denominator.

2. $f(x) = \frac{x+2}{x^2+5x+6}$ HA $y=0$

f(x) has a horizontal asymptote at $y=0$ b/c the degree of numerator < degree of the denominator.

3. $h(x) = \frac{-2x^2+5x-2}{2x^2+7x+3}$ HA $y=-\frac{2}{2}=-1$

h(x) has the same degree in the numerator and denominator.

∴ h(x) has a horizontal asymptote at $y=-\frac{2}{2}=-1$

4. $h(x) = \frac{2x^2+7x+6}{x^2+5x+6}$ HA $y=\frac{2}{1}=2$

h(x) has the same degree in the numerator and denominator.

∴ h(x) has a horizontal asymptote at $y=2$

5. $g(x) = \frac{x^2+3x+2}{x+1}$ No HA

g(x) has a slant asymptote b/c the degree of the numerator is one more than the degree of the denominator.

$$\begin{array}{r|rrr} 1 & 1 & 3 & 2 \\ & 0 & 1 & 4 \\ \hline & 1 & 4 & R6 \end{array}$$

g(x) has a slant asymptote at $y=x+4$

6. $g(x) = \frac{x^2-5x-6}{2x+4} = \frac{(x-6)(x+1)}{2(x+2)}$ No HA. CAUTION

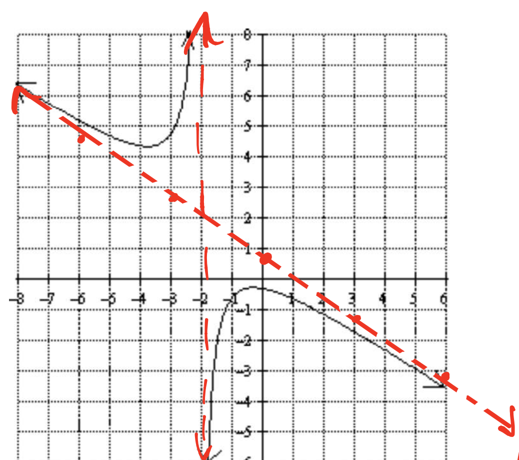
g(x) has a slant asymptote b/c the degree of the numerator is one more than the degree of the denominator.

[Must divide (x^2-5x-6) by (2) and $(x+2)$]

$$\begin{array}{r|rrr} -2 & 1 & -5 & -6 \\ & 0 & -2 & 14 \\ \hline & 1 & -7 & R8 \\ & \frac{}{2} & \frac{}{2} & \end{array}$$

∴ g(x) has a slant asymptote at $y=\frac{1}{2}x-\frac{7}{2}$

The graph of the function $g(x) = \frac{-2x^2-2x-2}{3x+6}$ is pictured. Find the equation(s) of the asymptotes of the graph of $h(x)$ and draw them on the graph.



$g(x) = \frac{-2x^2-2x-2}{3(x+2)}$

VA at $x=-2$

CAUTION! Divide by 3

$$\begin{array}{r|rrr} -2 & -2 & -2 & -2 \\ & 0 & 4 & -4 \\ \hline & -2 & 2 & R-6 \\ & \frac{}{3} & \frac{}{3} & \end{array}$$

g(x) has a slant asymptote at $y=-\frac{2}{3}x+\frac{2}{3}$

g(x) has a vertical asymptote at $x=-2$