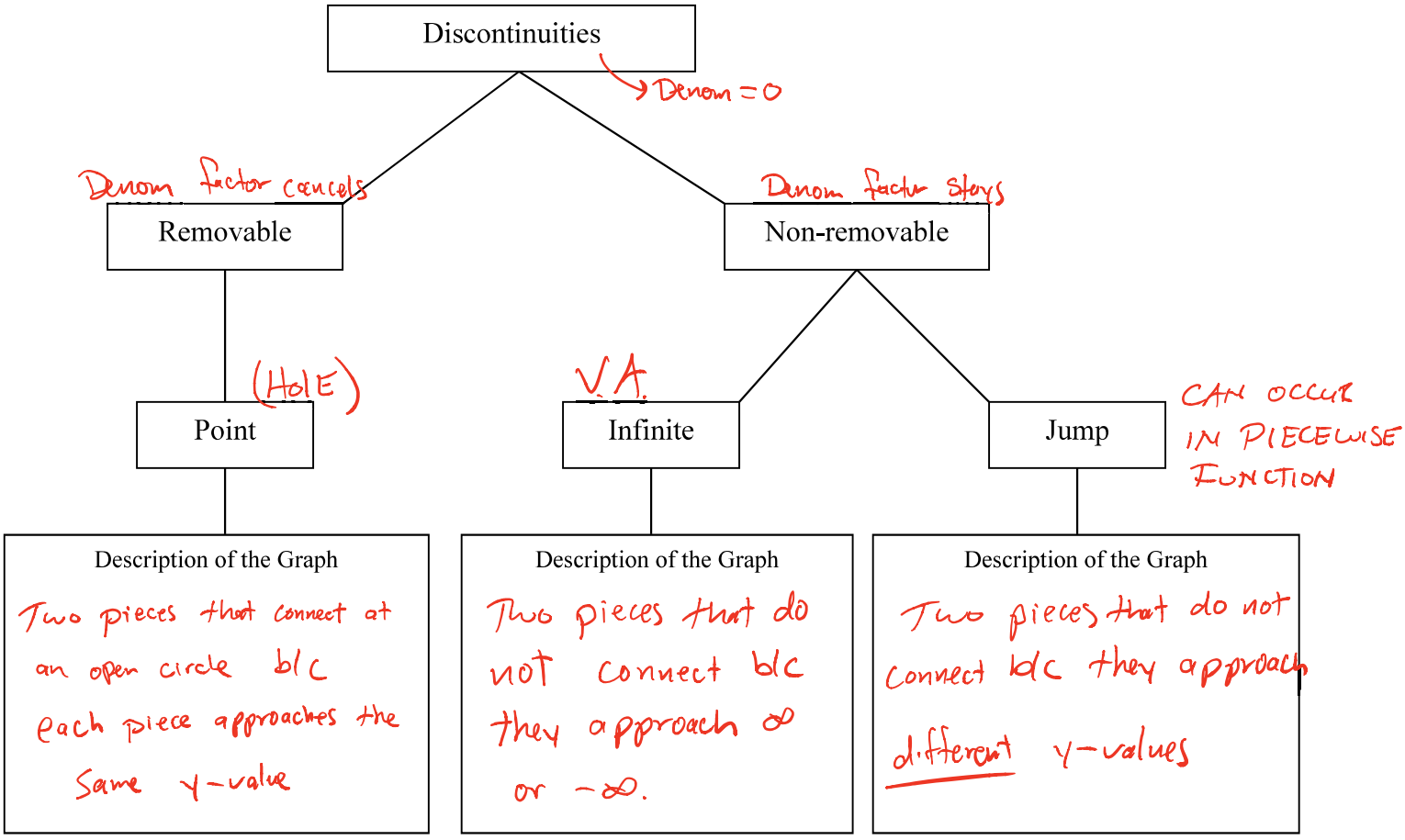
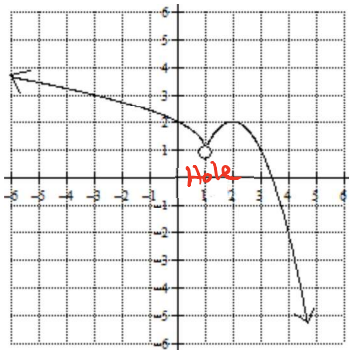
Notes 5.3 Discontinuities of Rational Functions

An Analytical Approach

There are two major types of discontinuities in functions.



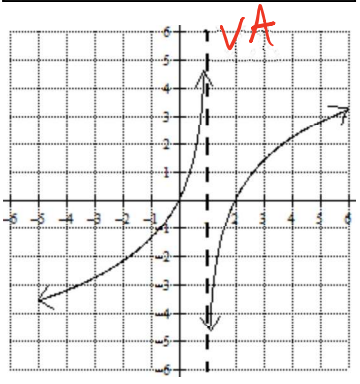
The graph below has a removable, point discontinuity at $x = 1$.



Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, \infty)$

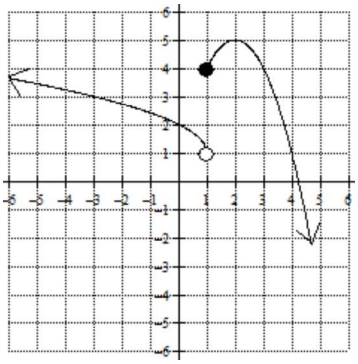
The graph below has an non-removable, infinite discontinuity at $x = 1$.



Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, \infty)$

The graph below has a non-removable, jump discontinuity at $x = 1$.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Find each of the indicated graphical properties: (a) values of the zero(s), (b) coordinates of the y-intercept, (c) equation(s) of the vertical asymptote, (d) coordinates of any holes on the graph. If a function does not have a particular property, explain why it does not. Show your work.

1. $f(x) = \frac{2x^2-5x-3}{x^2-x-6} = \frac{(2x+1)(x-3)}{(x+2)(x-3)}$ zero @ $x=-\frac{1}{2}$; y-int $=\frac{-3}{-6}$; VA @ $x=-2$; Hole @ $x=3$

$f(x) = \frac{2x+1}{x+2}$, hole $(3, \frac{7}{5})$

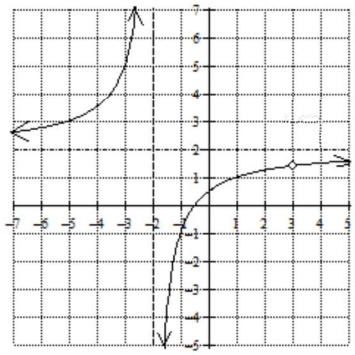
a) Zeros: $x=-\frac{1}{2}$
$2x+1=0$
$2x=-1$
$x=-\frac{1}{2}$

b) y-int $=(0,\frac{1}{2})$
y-int $=\frac{-3}{-6}$
y-int $=\frac{1}{2}$

c) VA: $x=-2$
$x+2=0$
$x=-2$

Holes $(3, \frac{7}{5})$

d)
$x-3=0$	$f(x)=\frac{2x+1}{x+2}$
$x=3$	$f(3)=\frac{2(3)+1}{(3)+2}$
	$=\frac{6+1}{5}$
	$f(3)=\frac{7}{5}$



2. $g(x) = \frac{x^2+4x+3}{x^2+2x-3} = \frac{(x+3)(x+1)}{(x+3)(x-1)}$ y-int $=\frac{3}{-3}$; zero @ $x=-1$; Hole @ $x=-3$; VA @ $x=1$

$g(x) = \frac{x+1}{x-1}$, hole $(-3, \frac{1}{2})$

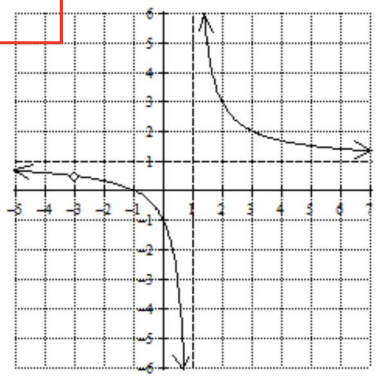
a) Zeros: $x=-1$
$x+1=0$
$x=-1$

b) y-int $=(0,-1)$
y-int $=\frac{3}{-3}$
$=-1$

c) VA: $x=1$
$x-1=0$
$x=1$

d) hole $(-3, \frac{1}{2})$

$x+3=0$	$g(-3)=\frac{-3+1}{-3-1}$
$x=-3$	$=\frac{-2}{-4}$
	$g(-3)=\frac{1}{2}$



3. $h(x) = \frac{x+2}{x^2+5x+6} = \frac{x+2}{(x+2)(x+3)}$ y-int $=\frac{2}{6}$; no zero; Hole @ $x=-2$; VA @ $x=-3$

$h(x) = \frac{1}{x+3}$, hole $(-2, 1)$

a) zeros: none b/c there is no non-canceling factor in the numerator

b) y-int $=\frac{2}{6}=\frac{1}{3}$ $(0,\frac{1}{3})$

c) VA: $x+3=0$
$x=-3$

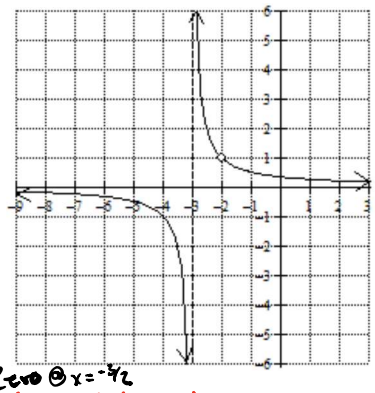
d) Hole: $x+2=0$
$x=-2$

$h(x)=\frac{1}{x+3}$

$h(-2)=\frac{1}{-2+3}$

$=\frac{1}{1}$

$h(-2)=1$



4. $p(x) = \frac{2x^2+7x+6}{x^2+5x+6} = \frac{(2x+3)(x+2)}{(x+3)(x+2)}$ zero @ $x=-\frac{3}{2}$; VA @ $x=-3$; Hole @ $x=-2$

$p(x) = \frac{2x+3}{x+3}$, hole $(-3,-1)$

a) zero: $x=-\frac{3}{2}$
$2x+3=0$
$2x=-3$
$x=-\frac{3}{2}$

b) y-int: $(0,1)$
y-int $=\frac{6}{6}$
y-int $=1$

c) VA: $x=-3$
$x+3=0$
$x=-3$

d) hole $(-3,-1)$

$x+2=0$	$p(x)=\frac{2x+3}{x+3}$
$x=-2$	$p(-2)=\frac{2(-2)+3}{(-2)+3}$
	$=\frac{-4+3}{1}$
	$=\frac{-1}{1}$
	$p(-2)=-1$