Notes 4.2 Finding Rational Roots of Polynomial Functions

The Rational Root Theorem

Consider the quadratic function $f(x) = 3x^2 + 17x - 6$, which is itself a polynomial function of degree 2. In the left hand boxes below, list all of the POSSIBLE pairs of factors that could be factors of the function. Then, in the right hand boxes below, list all of the POSSIBLE roots of the function based on the factors.

Possible Factors of *f*(*x*)	**Possible Roots of *f*(*x*)**
$(3x+1)(x-6)$	$x = -\frac{1}{3}$, $x = 6$
$(3x-1)(x+6)$	$x = \frac{1}{3}$, $x = -6$
$(3x-6)(x+1)$	$x = 2$, $x = -1$
$(3x+6)(x-1)$	$x = -2$, $x = 1$
$(3x+3)(x-2)$	$x = -1$, $x = 2$
$(3x-3)(x+2)$	$x = 1$, $x = -2$
$(3x+2)(x-3)$	$x = -2/3$, $x = 3$
$(3x-2)(x+3)$	$x = 2/3$, $x = -3$

From the equation of *f*(*x*), what are all of the factors of the constant term of the function?

$\pm 1, \pm 2, \pm 3, \pm 6$

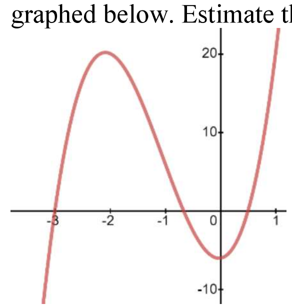
From the equation of *f*(*x*), what are all of the factors of the leading coefficient of the function?

$\pm 1, \pm 3$

Now, place each factor of the constant term over each factor of the leading coefficient and compare this list of values to the list of Possible Roots of *f*(*x*) from the right hand boxes above. What do you notice?

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/3, \pm 2/3,$

Now, consider the function $f(x) = 6x^3 + 19x^2 + x - 6$ graphed below. Estimate the roots of the function.



Root likely
$x = -3$, $x = -2/3$ and $x = \frac{1}{2}$

The problem with this graph is that one of the roots, $x = -3$, seems clearly visible from the graph but the other two roots are visible but because they are not whole number values, it is impossible to tell what the values are. Thus, we have what is called the Rational Root Theorem to help us out. The Rational Root Theorem provides a list of all the POSSIBLE rational values that could be roots of the function using a manner similar to the investigation of the quadratic function at the beginning of the lesson.

The Rational Root Theorem

A list of all rational numbers that are possible roots of a polynomial function can be formed by placing each integer factor of the constant term over each integer factor of the leading coefficient.

Now, apply the Rational Root Theorem to list all of the possible values that could be roots of $f(x) = 6x^3 + 19x^2 + x - 6$.

Possible Rational Roots $= \dfrac{\text{Factors of } -6}{\text{Factors of } 6} = \dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 3, \pm 6}$

$= \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm 3/2, \pm \frac{1}{3}, \pm 2/3, \pm 1/6$

From the list above, eliminate those values that do not appear to be the values indicated on the graph. Which values from the list do appear to be roots? What can be done to verify that they are, in fact, roots of the function? Show that work below.

Highly Possible Rational Roots $= -3, -2/3, \frac{1}{2}$

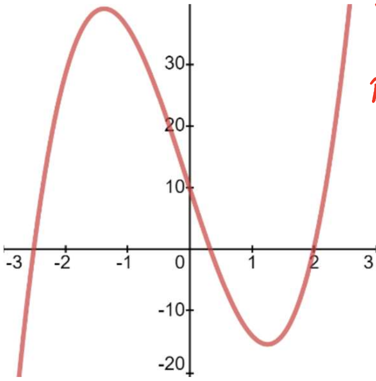
$\boxed{-3}$	6	19	1	−6
		−18	−3	6
	6	1	−2	0

$6x^2 + x - 2 = 6x^2 + 4x - 3x - 2$
$= 2x(3x+2) - 1(3x+2)$
$= (3x+2)(2x-1)$

$\boxed{\text{Roots: } -3, -2/3, \frac{1}{2}}$

Use the rational root theorem to determine the rational roots for each of the functions below. Use synthetic division to verify that they are, in fact, roots.

1. $g(x) = 6x^3 + x^2 - 31x + 10$



$PRR = \dfrac{\text{Factors } 10}{\text{Factors } 6}$

$PRR = \dfrac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2, \pm 3, \pm 6}$

$HPRR = -5/2, \frac{1}{3}, 2$

$$\begin{array}{c|cccc} \boxed{2} & 6 & 1 & -31 & 10 \\ & 0 & 12 & 26 & -10 \\ \hline & 6 & 13 & -5 & \end{array}$$

$6x^2 + 13x - 5$

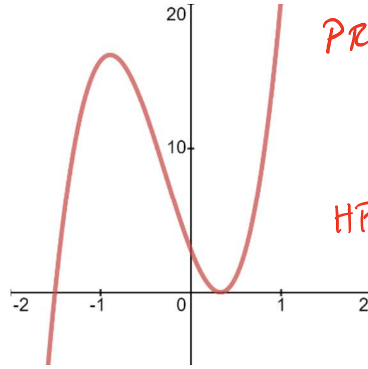
$= 6x^2 + 15x - 2x - 5$

$= 3x(2x+5) - 1(2x+5)$

$= (2x+5)(3x-1)$

$\boxed{\text{Roots: } -5/2, \frac{1}{3}, 2}$

2. $f(x) = 18x^3 + 15x^2 - 16x + 3$



$PRR = \dfrac{\text{Factors } 3}{\text{Factors } 18}$

$= \dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18}$

$HPRR = -\frac{3}{2}, \frac{1}{3}$

$$\begin{array}{c|cccc} \boxed{\frac{1}{3}} & 18 & +15 & -16 & 3 \\ & 0 & 6 & 7 & -3 \\ \hline \boxed{\frac{1}{3}} & 18 & 21 & -9 & \boxed{0} \\ & 0 & 6 & 9 & \\ \hline & 18 & 27 & \boxed{0} & \end{array}$$

$18x + 27 = 0$

$18x = -27$

$x = \frac{-27}{18}$

$x = -3/2$

$\boxed{\text{Roots: } -3/2, \frac{1}{3}, \frac{1}{3}}$