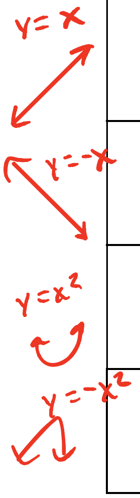
Notes 3.6 Graphs of Polynomial Functions

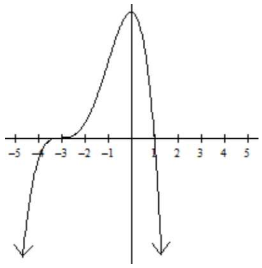
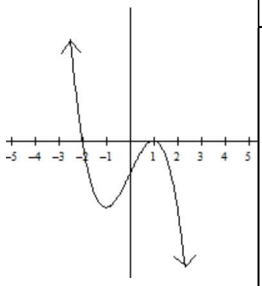
Analytical, Graphical and Numerical Approaches

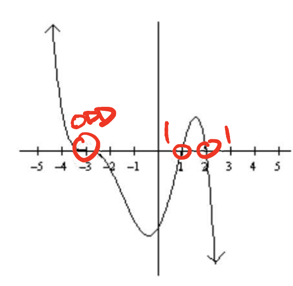
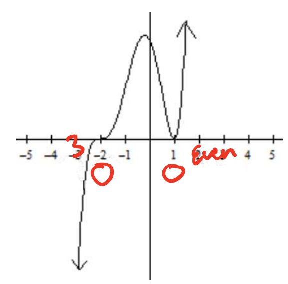
Complete the following table, identifying the end behavior rules for any odd and even degreed polynomial function based on the degree of the function and the leading coefficient.



	Degree	Leading Coefficient	Left End Behavior	Right End Behavior
1.	ODD	+	$\lim_{x\to -\infty} f(x) = -\infty$	$\lim_{x\to \infty} f(x) = \infty$
2.	ODD	—	$\lim_{x\to -\infty} f(x) = \infty$	$\lim_{x\to \infty} f(x) = -\infty$
3.	EVEN	+	$\lim_{x\to -\infty} f(x) = \infty$	$\lim_{x\to \infty} f(x) = \infty$
4.	Even	-	$\lim_{x\to -\infty} f(x) = -\infty$	$\lim_{x\to \infty} f(x) = -\infty$

Complete the table below for each graphed function.

Graph of the Function	**Zeros and Their Multiplicities**		**Degree and Type of Function**	**Determine if the leading coefficient of the equation is positive or negative and justify your conclusion. Then, write a possible equation for the function.**
	Zeros	Multiplicity		
	x=-3 x=1	ODD ≥3 ODD=1	Even Degree ≥4	Function is even degree with both end behaviors falling. ∴ Lead coefficent is negative. $f(x) = -a(x+3)^3(x-1)$
	x=-2 x=1	ODD =1 Even ≥2	ODD DEGREE ≥3	Function is ODD degree, rises to left and falls to right. ∴ Lead coefficient is negative $f(x) = -a(x+2)(x-1)^2$

<table>
<tr><th rowspan="2">Graph of the Function</th><th colspan="2">Zeros and Their Multiplicities</th><th rowspan="2">Degree and Type of Function</th><th rowspan="2">Determine if the leading coefficient of the equation is positive or negative and justify your conclusion. Then, write a possible equation for the function.</th></tr>
<tr><th>Zeros</th><th>Multiplicity</th></tr>
<tr><td> </td><td>x=-3
x=-1
x=2</td><td>ODD ≥3
ODD=1
ODD=1</td><td>ODD DEGREE ≥ 5</td><td>Function is ODD degree, rises to left and falls to right.
∴ Lead coefficient is negative
$f(x)=-a(x+3)^3(x+1)(x-2)$</td></tr>
<tr><td></td><td>Zeros</td><td>Multiplicity</td><td></td><td></td></tr>
<tr><td> </td><td>x=-2
x=1</td><td>ODD=1
EVEN≥2</td><td>ODD Degree ≥3</td><td>Function is ODD degree, falls to left and rises to right.
∴ Lead coefficient is positive
$f(x)=a(x+2)(x-1)^2$</td></tr>
</table>

The table below shows function values of a quintic polynomial function. The zeros in the table are the only zeros of the function. Answer the questions that follow.

mult ODD   mult even

x	–6	–3.5	–3	–2.5	0.5	1	1.5	6
$F(x)$	–147	–10.125	0	6.125	0.875	0	1.125	225

a. Identify the left and right end behavior of the function, $F(x)$.

$$\lim_{x\to-\infty}F(x)=-\infty \qquad \lim_{x\to\infty}F(x)=\infty$$

b. Based on the end behavior, what must be true about the degree of the function and about the sign of the leading coefficient of the equation? Give a reason for your answer.

Since the end behaviors are opposite of each other, the degree is ODD. Since the graph falls to the left and rises to the right, the lead coefficient is positive.

c. Identify the zeros and the multiplicities of each zero of $F(x)$ Justify your answers.

- F(x) has zeros at x=-3 and x=1 b/c F(x)=0.
- F(x) has a zero at x=-3 with ODD multiplicity b/c F(x) changes signs at x=-3 and therefore F(x) crosses the x-axis at x=-3.
- F(x) has a zero at x=1 with EVEN multiplicity b/c F(x) doesn't change signs at x=1 and therefore F(x) is tangent at x=1.

The table below shows function values of a polynomial function. The zeros in the table are the only zeros of the function, and no zeros have a multiplicity greater than two. Answer the questions that follow.

x	–6	–3.5	–3	–2.5	–1.5	–1	–0.5	1.5	2	2.5	4
$G(x)$	360	3.437	0	1.688	3.9375	0	–7.8123	–25.313	0	52.938	490
			Even=2			ODD=1			ODD=1		

a. Identify the left and right end behavior of the function, $G(x)$.

$\lim_{x\to -\infty} G(x) = \infty$ $\lim_{x\to \infty} G(x) = \infty$

b. Based on the end behavior, what must be true about the degree of the function and about the sign of the leading coefficient of the equation? Give a reason for your answer.

Since both end behaviors rise, the degree of G(x) is even with positive lead coefficient.

c. Identify zeros and the multiplicities of each zero of $G(x)$ Justify your answers.

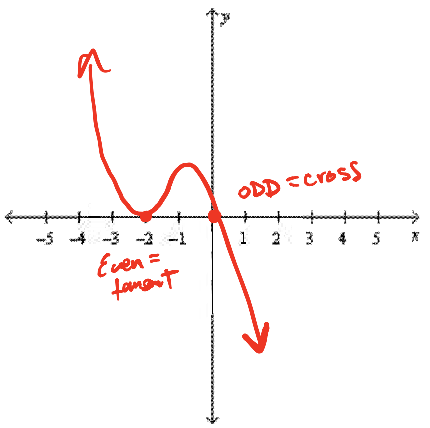
- G(x) has zeros at $x = -3, -1$ and 2 b/c $G(x) = 0$.
- G(x) has zeros at $x = -1$ and $x = 2$ each with ODD multiplicity of 1 b/c G(x) changes signs at $x = -1$ and $x = 2$ AND no zero has multiplicity greater than 2.
- G(x) has a zero at $x = -3$ with even multiplicity of 2 b/c G(x) doesn't change signs at $x = -3$ AND no zero has multiplicity greater than 2.

If the given function is not already factored, completely factor each of the functions below, identifying their zeros and the multiplicity of those zeros. Then, using the zeros and the end behavior, sketch a graph of the function on the axes provided.

$F(x) = -2x^3 - 8x^2 - 8x$

$F(x) = -2x(x^2 + 4x + 4)$

$F(x) = -2x(x+2)^2$



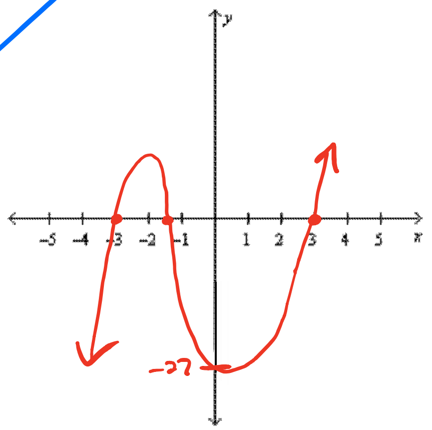
$G(x) = \underbrace{2x^3 + 3x^2} - \underbrace{18x - 27}$

$G(x) = x^2(2x+3) - 9(2x+3)$

$= (2x+3)(x^2-9)$

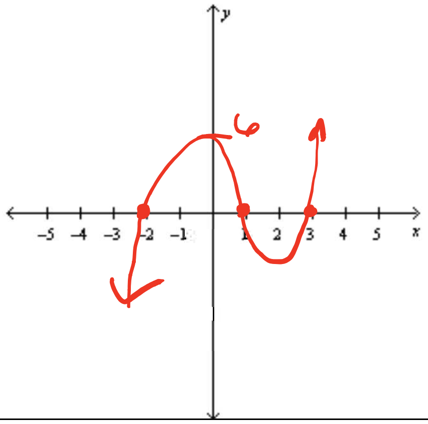
$= \underline{2}(x + \frac{3}{2})(x-3)(x+3)$

why ?



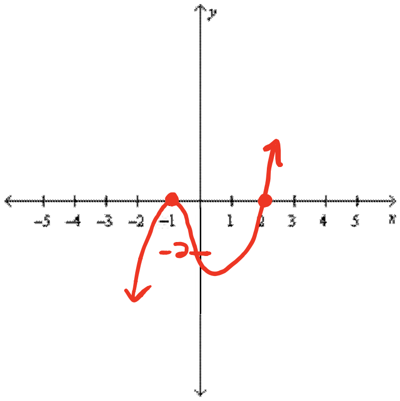
$f(x) = (x + 2)(x - 1)(x - 3)$

$f(x) = x^3 + \cdots + 6$



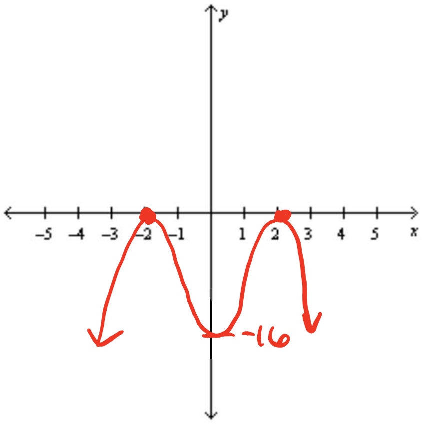
$g(x) = (x + 1)^2(x - 2)$

$g(x) = x^3 + \cdots - 2$



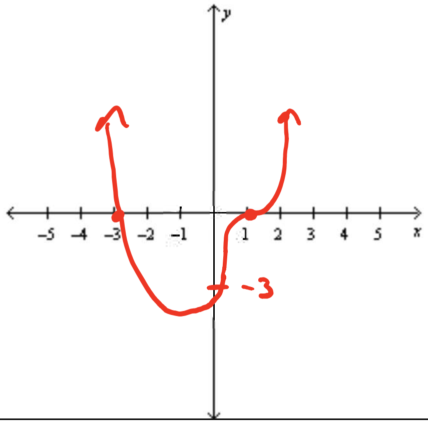
$h(x) = -(x + 2)^2(x - 2)^2$

$h(x) = -x^4 + \cdots - 16$



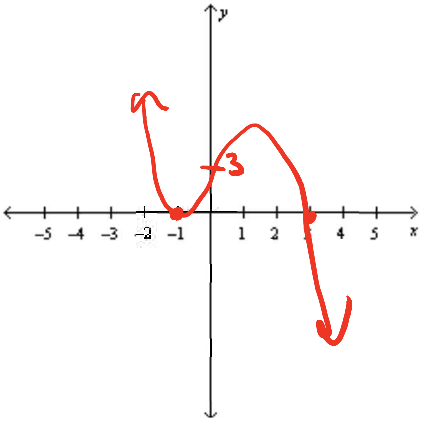
$k(x) = (x + 3)(x - 1)^3$

$k(x) = x^4 + \cdots - 3$



$g(x) = (x + 1)(x + 1)(3 - x)$

$g(x) = -x^3 + \cdots + 3$



$k(x) = (x + 3)(3 - 2x)(x - 3)^2$

$= (x+3)(-2)(x - 3/2)(x-3)^2$

$k(x) = -2x^4 + \cdots + 81$