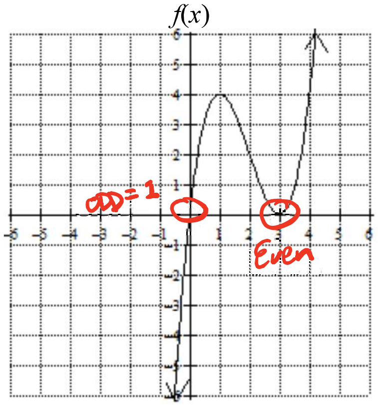
Notes 3.5 Properties of Graphs of Polynomial Functions

Terminology Associated with Graphs of Polynomial Functions

Determine what types of polynomial functions *f*, *g*, and *h* are graphed below. Give a reason for your conclusions based on the zeros of each function.

f(*x*)



• f(x) cross the x-axis @ x=0 and doesn't change concavity

∴ x=0 has ODD multiplicity of 1

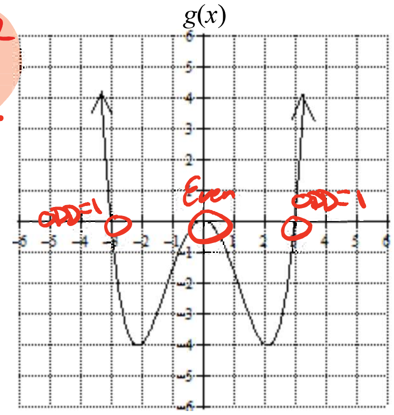
• f(x) is tangent to x-axis @ x=3

∴ x=3 has EVEN multiplicity of 2 or more.

∴ f(x) is of odd degree and at least cubic.

online only

g(*x*)



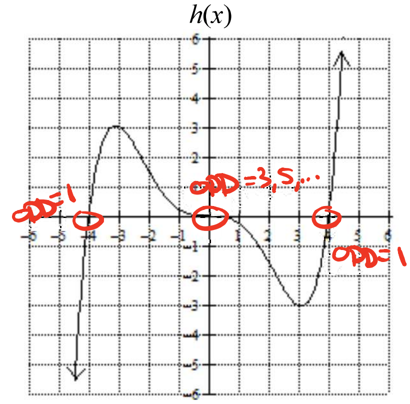
• g(x) cross the x-axis @ x=-3 and x=3 and doesn't change concavity ∴ x=-3 and x=3 have ODD multiplicity of 1

g(x) is tangent to x-axis @ x=0

∴ x=0 has EVEN multiplicity of 2 or more.

∴ g(x) is of even degree of at least quartic.

h(*x*)



• h(x) cross the x-axis @ x=-4 and x=4 and doesn't change concavity ∴ x=-4 and x=4 have ODD multiplicity of 1.

• h(x) cross the x-axis @ x=0 and changes concavity ∴ x=0 has ODD multiplicity of 3 or more.

∴ h(x) is of ODD degree and at least Quintic

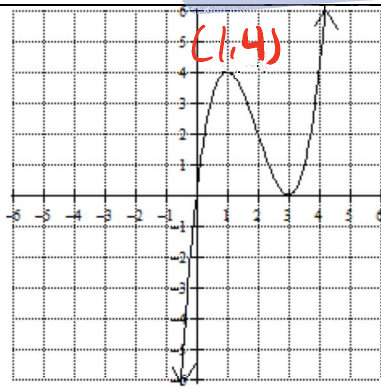
As we begin to study the graphs of polynomial functions, there are other properties of graphs that now can be identified. For the remainder of this lesson, we will define terms and then we will identify these terms on from the graphs of the given functions.

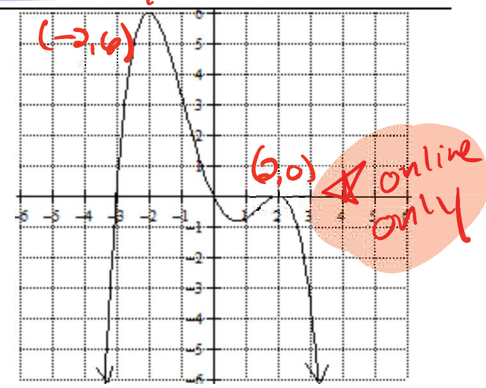
Extreme values: MAX / MIN

(aka local)

Relative Maximum: The point where the graph changes from increasing to decreasing.

Label and identify the relative maximum(s) on the graphs of the functions pictured to the right.

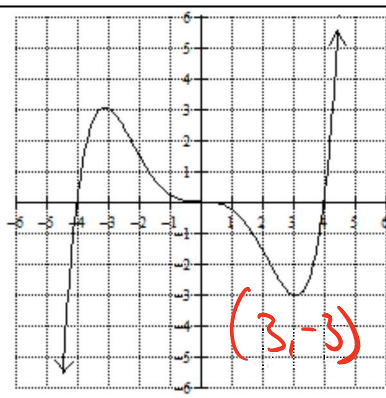


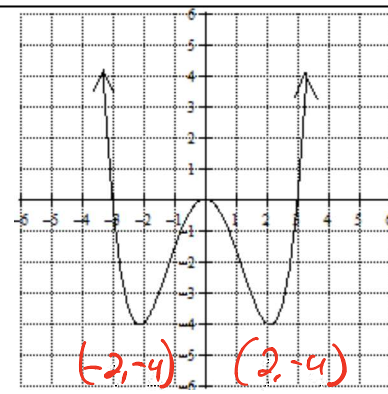


(aka local)

Relative Minimum: The point where the graph changes from decreasing to increasing.

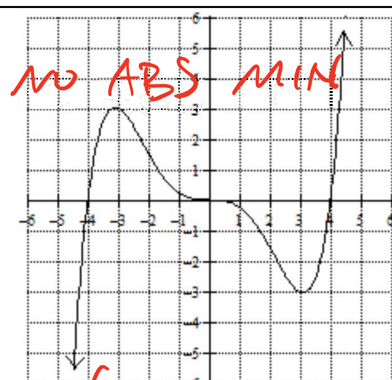
Label and identify the relative minimum(s) on the graphs of the functions pictured to the right.

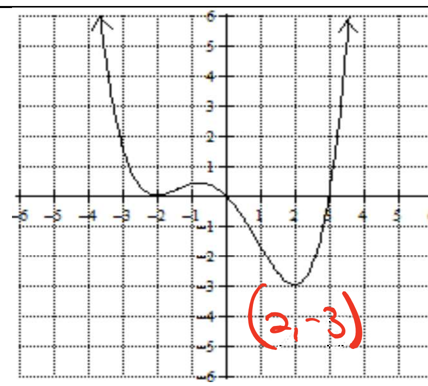




Absolute Minimum: The point on the graph that has the lowest y-value.

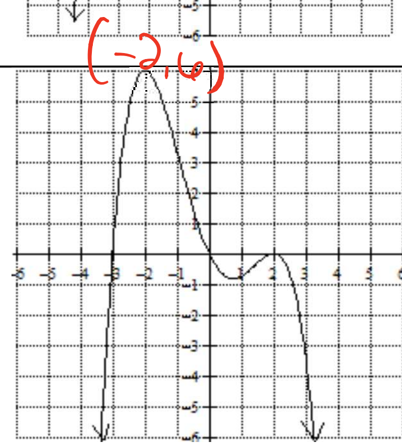
Label and identify the absolute minimum(s) on the graphs of the functions pictured to the right.

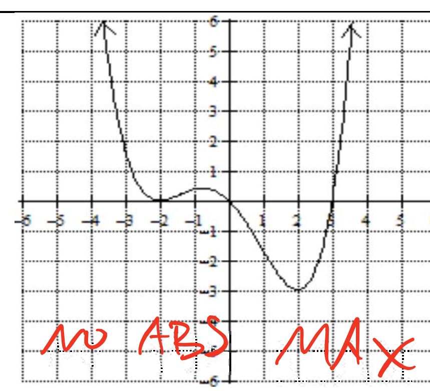




Absolute Maximum: The point on the graph that has the highest y-value.

Label and identify the absolute maximum(s) on the graphs of the functions pictured to the right.





Point of Inflection: A point where the graph changes concavity.

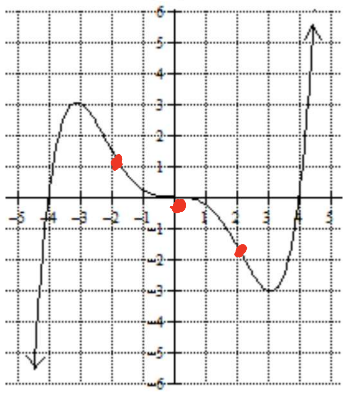
Intervals of Concavity: Open intervals along the x-axis where a graph is concave up or down.

A point of inflection can occur at 1 of 3 places:

1. At a zero with ODD multiplicity ≥ 3
2. Between a zero of ODD multiplicity ≥ 3 and the nearest local max/min (the Point of Inflection is approximately halfway between those points)
3. Between consecutive extreme values. (the Point of Inflection is approximately halfway between those points)

1. How many points of inflection does the graphed function below have? Label their approximate locations on the graph.

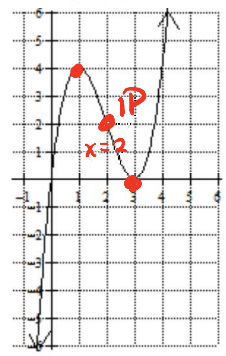
$f(x)$



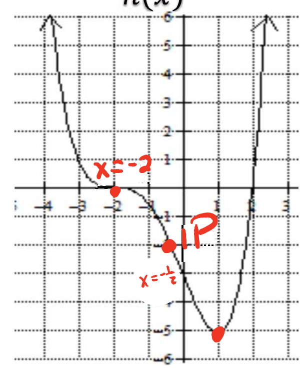
3 P.I

2. Use the graphs below to answer each question.

$g(x)$



$h(x)$



a. Approximate the x-coordinates of all points of inflection for the graphed functions below and label the point on the graph

g(x)	h(x)
PoI: $x \approx \frac{1+3}{2} \approx \frac{4}{2} \approx 2$	PoI: $x = -2$ $x \approx \frac{-2+1}{2} \approx -\frac{1}{2}$

b. Identify the interval(s) on which the graphed functions to the right are concave up & concave down.

g(x)

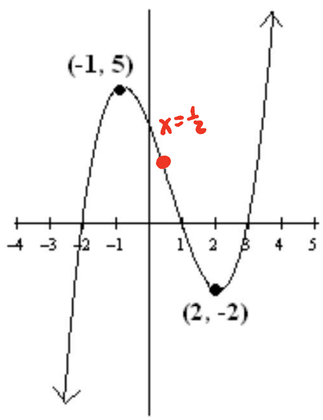
$\boxed{\text{Concave up } (2,\infty) \quad \text{Concave Down } (-\infty,2)}$

h(x)

$\boxed{\text{concave up } (-\infty,-2)\cup(-\tfrac{1}{2},\infty) \quad \text{concave down } (-2,-\tfrac{1}{2})}$

4. Use the graph below to answer each question.

Graph of $g(x)$



a. Zeros and their multiplicities, with reasoning:

x = -2, 1 and 3 each have ODD multiplicity of 1 because g(x) cross x-axis at x=-2, 1 and 3 doesn't change concavity.

b. Type of Function and why:

f(x) is an ODD degree of at least 3 b/c the sum of multiplicities is at least 3.

c. Relative Maximum(s):

(-1,5)

d. Relative Minimum(s):

(2,-2)

e. Absolute Maximum(s):

None

f. Absolute Minimum(s):

None

g. Approximate x-value(s) of Points of Inflections:

PoI: $x \approx \frac{-1+2}{2} \approx \frac{1}{2}$

h. Intervals of x – values where $g(x)$ is concave up:

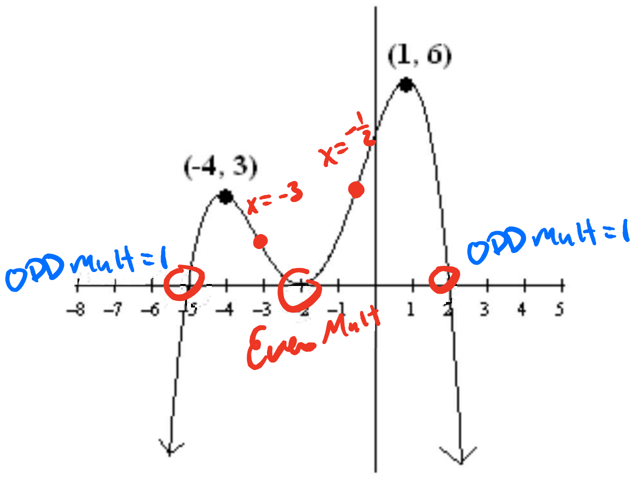
$(\frac{1}{2}, \infty)$

i. Intervals of x – values where $g(x)$ is concave down:

$(-\infty, \frac{1}{2})$

5. Use the graph below to answer each question.

Graph of $h(x)$



a. Left End Behavior:

$\lim_{x \to -\infty} f(x) = -\infty$

b. Right End Behavior:

$\lim_{x \to \infty} f(x) = -\infty$

c. Zeros and their multiplicities, with reasoning:

• h(x) has zeros at $x=-5$ and $x=2$ with ODD mult of 1 b/c h(x) crosses the x-axis at $x=-5$ and $x=2$ and doesn't change concavity.

• h(x) has zero at $x=-2$ with even mult b/c h(x) is tangent to x-axis at $x=-2$.

d. Type of Function and why:

h(x) is even degree of 4 or more because the sum of the multiplicities is at least 4.

e. Relative Maximum(s):

$(-4, 3)$, $(1, 6)$

f. Relative Minimum(s):

$(-2, 0)$

g. Absolute Maximum(s):

$(1, 6)$

h. Absolute Minimum(s):

None

i. Approximate x-value(s) of Points of Inflections:

PoI

$x \approx \frac{-4+(-2)}{2} = \frac{-6}{2} = -3$

$x \approx \frac{-2+1}{2} = \frac{-1}{2}$

j. Intervals of x – values where $g(x)$ is concave up:

$(-3, -\frac{1}{2})$

k. Intervals of x – values where $g(x)$ is concave down:

$(-\infty, -3) \cup (-\frac{1}{2}, \infty)$