Notes 3.3 The Remainder Theorem of Polynomial Functions

Remainder Theorem

If a polynomial function, $f(x)$, is divided by $(x-a)$, then the remainder of $\frac{f(x)}{x-a}$ is $f(a)$

Quick review of synthetic division…

$(2x^3 + x^2 - 13x + 6) \div (2x - 1)$

$2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$

$\frac{1}{2}$	2	1	-13	6
	0	1	1	-6
	2	2	-12	0

$\therefore \frac{2x^3+x^2-13x+6}{2x-1} = 2x^2+2x-12$

Does the Remainder Theorem really work?

For each of the functions in the table below, use the graphing calculator to determine the given function value. Then, use synthetic division to determine the remainder when $P(x) = 2x^4 + 7x^3 + 4x^2 - 7x - 6$ is divided by the given divisor.

P(a)	Divisor	Remainder when $P(x)$ is divided by the given divisor.
$P(1) =$ 0	$(x-1)$ $x-1=0$ $x=1$	1 | 2 7 4 -7 -6 0 2 9 13 6 2 9 13 6 | 0
$P(2) =$ 84	$(x-2)$ $x-2=0$ $x=2$	2 | 2 7 4 -7 -6 0 4 22 52 90 2 11 26 45 | 84
$P(-3) =$ 24	$(x+3)$ $x+3=0$ $x=-3$	-3 | 2 7 4 -7 -6 0 -6 -3 -3 30 2 1 1 -10 | 24
$P\left(-\frac{3}{2}\right) =$ 0	$(2x+3)$ $2x+3=0$ $2x=-3$ $x=-3/2$	-3/2 | 2 7 4 -7 -6 0 -3 -6 3 6 2 4 -2 -4 | 0

1. For what value of k will the function $P(x) = -2x^3 - 2x^2 + kx - 2$ have a remainder of 8 when divided by the factor $(x + 2)$? $= 0$

$x = -2$

$\boxed{-2}$	-2	-2	k	-2
	0	4	-4	$-2k+8$
	-2	2	$(k-4)$	$-2k+6$

$-2k+6 = 8$

$-2k = 2$

$k = -1$

2. For what value of k will the function $P(x) = -x^4 - 2x^2 + kx - 6$ have a remainder of 0 when divided by the factor $(x + 1)$? $= 0$

$x = -1$

$\boxed{-1}$	-1	0	-2	k	-6
	0	1	-1	3	$-k-3$
	-1	1	-3	$(k+3)$	$-k-9$

$-k-9 = 0$

$-9 = k$

3. Suppose the function $g(x) = 3x^2 - 2x^3 + 3x - 2$ is divided by the factor $(x - 2)$. Which is greater: the value of $g(-2)$ or the remainder when $g(x)$ is divided by $(x - 2)$? Show your work.

$\boxed{-2}$	-2	3	3	-2
	0	4	-14	22
	-2	7	-11	20

$g(-2) = 20$

$\boxed{2}$	-2	3	3	-2
	0	-4	-2	2
	-2	-1	1	0

Remainder $= 0$

$g(-2)$ is greater than the remainder when $g(x)$ is divided by $(x-2)$

Factor Theorem

If a polynomial function, $f(x)$, is divided by on if its factors $(x - a)$, then the remainder of $\frac{f(x)}{x-a}$ is $f(a) = 0$

TL:DR $(x-a)$ is a factor $\therefore$ $x = a$ is a zero

The Factor Theorem of Polynomial Functions

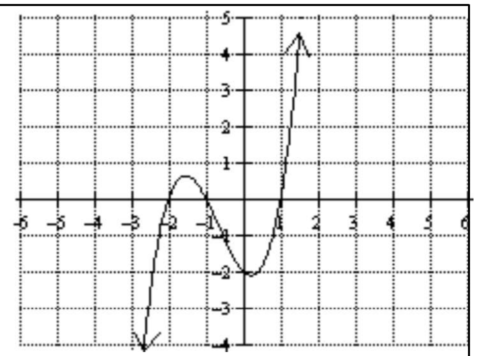
The equations of the functions below are given in both factored form and expanded form, and a graph of the function is given, as well. From the graph, identify the zeros of the function. Then, synthetically divide the expanded form of the function by each factor.

1. $f(x) = (x + 2)(x - 1)(x + 1)$ $\quad f(x) = x^3 + 2x^2 - x - 2$

$$\boxed{-2}\quad \begin{array}{cccc} 1 & 2 & -1 & -2 \\ 0 & -2 & 0 & 2 \\ \hline 1 & 0 & -1 & \boxed{0} \end{array}$$

$$\boxed{-1}\quad \begin{array}{cccc} 1 & 2 & -1 & -2 \\ 0 & -1 & -1 & 2 \\ \hline 1 & 1 & -2 & \boxed{0} \end{array}$$

$$\boxed{-1}\quad \begin{array}{cccc} 1 & 2 & -1 & -2 \\ 0 & -1 & -1 & 2 \\ \hline 1 & 1 & -2 & \boxed{0} \end{array}$$

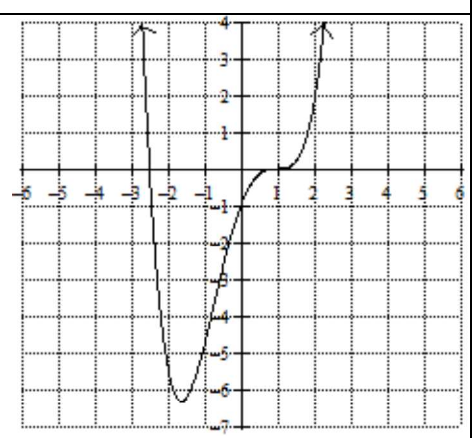


$x = -2, -1, 1$

2. $p(x) = 0.2(2x + 5)(x - 1)^3$ $\quad p(x) = 0.4x^4 - 0.2x^3 - 1.8x^2 + 2.6x - 1$

$$\boxed{-2.5}\quad \begin{array}{ccccc} 0.4 & -0.2 & -1.8 & 2.6 & -1 \\ 0 & -1 & 3 & -3 & 1 \\ \hline 0.4 & -1.2 & 1.2 & -0.4 & \boxed{0} \end{array}$$

$$\boxed{1}\quad \begin{array}{ccccc} 0.4 & -0.2 & -1.8 & 2.6 & -1 \\ 0 & 0.4 & 0.2 & -1.6 & 1 \\ \hline 0.4 & 0.2 & -1.6 & 1.0 & \boxed{0} \end{array}$$



$x = -2.5, 1$

Answer the following questions about factors and synthetic division.

1. Is $(2x + 1)$ a factor of $g(x) = 2x^3 + 3x^2 - 11x - 6$? Show and explain your work. What is the graphical interpretation of the result of this work?

$2x+1=0$
$2x=-1$
$x=-\frac{1}{2}$

$\boxed{-\frac{1}{2}}$	2	3	-11	-6
	0	-1	-1	6
	2	2	-12	$\boxed{0}$

$g(x) \div (2x+1)$ has a remainder of 0
∴ By the Factor Theorem, $(2x+1)$ is a factor of $g(x)$
∴ $x = -\frac{1}{2}$ is a zero

Graphically, $g(x)$ is on the x-axis at $x = -\frac{1}{2}$.

2. Consider the function $f(x) = x^3 - 13x + 12$. Divide $f(x)$ by $(x - 3)$ and then factor the quotient. Based on your work, what are the zeros of $f(x)$?

$x-3=0$
$x=3$

$\boxed{3}$	1	0	-13	12
	0	3	9	-12
	1	3	-4	$\boxed{0}$

By the Factor Theorem,
The zeros of $f(x)$ are $x = -4, 1, 3$

$f(x) = (x-3)(x^2+3x-4)$
$f(x) = (x-3)(x+4)(x-1)$

3. If $(x - 1)$ is a factor of $g(x) = 2x^3 - 3x^2 - 2x + 3$, what are the other two factors of $g(x)$ and rewrite $g(x)$ in completely factored form.

$\boxed{1}$	2	-3	-2	3
	0	2	-1	-3
	2	-1	-3	$\boxed{0}$

$g(x) = (x-1)(2x^2 - x - 3)$
$= (x-1)[\underline{2x^2 - 3x} + \underline{2x - 3}]$
$= (x-1)[x(2x-3) + 1(2x-3)]$
$g(x) = (x-1)(2x-3)(x+1)$

The other two factors of $g(x)$ are $(2x-3)$ and $(x+1)$

4. For what value of k is the factor $(x - 2)$ a factor of the function $p(x) = 6x^3 - 19x^2 + kx + 6$?

$\boxed{2}$	6	-19	k	6
	0	12	-14	2k-28
	6	-7	k-14	$\boxed{2k-22}$

$2k-22=0$
$2k=22$
$k=11$

$(x-2)$ is a factor of $p(x)$ when $k=11$

5. Is $(x + 1)$ is a factor of $f(x) = -3x^4 - 7x^3 - 3x^2 + 3x + 2$? If so, how many times is it a factor?

✓ $\boxed{-1}$	-3	-7	-3	3	2
	0	3	4	-1	-2
✓ $\boxed{-1}$	-3	-4	1	2	$\boxed{0}$
	0	3	1	-2	
✓ $\boxed{-1}$	-3	-1	2	$\boxed{0}$	
	0	3	-2		
X $\boxed{-1}$	-3	2	$\boxed{0}$		
	0	3			
	-3	$\boxed{5}$			

$f(x) = (x+1)^3(-3x+2)$.

• Yes, $(x+1)$ is a factor of $f(x)$.

• $(x+1)$ is a factor of $f(x)$ three times.