Notes 3.1 Factoring Cubic, Quartic, and Quintic Polynomials

The number one rule of factoring is that before anything is done to the polynomial, ***the terms must be ordered from greatest to least degree***. Beyond that, there is a series of things to check for to break the polynomial down into prime factors.

Four Terms

1) Factor out GCF
$aw \pm ax \pm ay \pm az = a[w \pm x \pm y \pm z]$

2) Factor by grouping

Factor $x^3 + 3x^2 - x - 3$

$= x^2(x+3) - 1(x+3)$

$= (x+3)(x^2-1)$

$= (x+3)(x-1)(x+1)$

Factor $2x^3 + x^2 - 18x - 9$

$= x^2(2x+1) - 9(2x+1)$

$= (2x+1)(x^2-9)$

$= (2x+1)(x-3)(x+3)$

Binomial

1) Factor out GCF
$ax \pm ay = a[x \pm y]$

2) Difference of Two Squares
$a^2 - b^2 = (a+b)(a-b)$

3) Difference/Sum of Two Cubes
$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Factor $2x^3 - 32x$

$= 2x(x^2 - 16)$

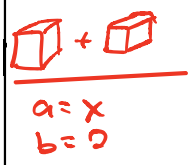
$= 2x(x-4)(x+4)$

□ - □

$a = x$
$b = 4$

Factor $2x^4 + 16x = 2x(x^3 + 8)$

$= 2x(x+2)(x^2 - 2x + 4)$



$a = x$
$b = 2$

Trinomials

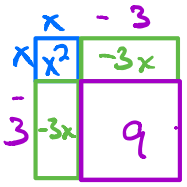
1) Factor out GCF
$ax \pm ay \pm az = a[x \pm y \pm z]$

2) Perfect Square Trinomial
$a^2 \pm 2ab + b^2 = (a \pm b)^2$

3) Splitting the Middle Term

Factor $x^2 - 6x + 9$ $= (x-3)^2$

Perfect □ tri?
$a = x$
$b = 3$
$2ab = 2 \cdot 3x$



Perfect □ tri?
$a = \sqrt{2}x^2$
$b = \sqrt{20}$
$2ab = 2\sqrt{40}x^2$
NADA

Factor $2x^4 - 13x^2 + 20$

$= 2x^4 - 8x^2 - 5x^2 + 20$

$= 2x^2(x^2-4) - 5(x^2-4)$

$= (x^2-4)(2x^2-5)$

$= (x-2)(x+2)(2x^2-5)$

$M = 2x^4(20)$
$A = -13x^2$
$N = -8x^2, -5x^2$

Factor each polynomial.

#1) $-2 + x^3 - x^2 + 2x$

$= x^3 - x^2 + 2x - 2$

$= x^3(x-1) + 2(x-1)$

$= (x-1)(x^3+2)$

#2) $-192x^2y - 72x^3 + 24rxy + 9rx^2$

$-3x[64xy + 24x^2 - 8ry - 3rx]$

$= -3x[8x(8y+3x) - r(8y+3x)]$

$= -3x(8y+3x)(8x-r)$

#3) $200 - 98x^2$

$= 2(100 - 49x^2)$

$= 2(10-7x)(10+7x)$

#4) $49x^2 - 100$

$= (7x-10)(7x+10)$

#5) $49x(x+4) - 100(x+4)$

$(x+4)(49x-100)$

#6) $x^2(x-10) + 17(x-10)$

$(x-10)(x^2+17)$

#7) $10x^2 + 100x + 250$

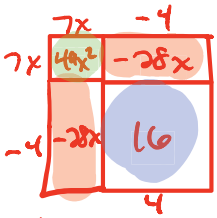
$= 10(x^2 + 10x + 25)$

$= 10(x+5)^2$

#8) $49x^2 - 56x + 16$

$= (7x-4)^2$

Tri?
1) Yes, $7x$
2) Yes, 4
3) $2 \cdot 7x \cdot 4 = 56x$ middle term. Yes



#9) $19x + 5x^2 + 12$

$= 5x^2 + 19x + 12$

$= 5x^2 + 4x + 15x + 12$

$= x(5x+4) + 3(5x+4)$

$= (5x+4)(x+3)$

$M = 5x^2(12) = 60x^2$

$A = 19x$

$N = 4x, 15x$

$60x^2$
$1x, 60x$
$2x, 30x$
$3x, 20x$
$4x, 15x$

#10) $-16x^2 - 60x + 100$

$= -4(4x^2 + 15x - 25)$

$= -4(4x^2 - 5x + 20x - 25)$

$= -4[x(4x-5) + 5(4x-5)]$

$= -4(4x-5)(x+5)$

$M = 4x^2(-25) = -100x^2$

$A = 15x$

$N = -5x, 20x$

$\frac{-100x^2}{-1, 100x}$
$-2x, 50x$
$-4x, 25x$
$-5x, 20x$

#11) $1029x^3y - 24y^4$

$= 3y(343x^3 - 8y^3)$

$= 3y(7x-2y)(49x^2 + 14xy + 4y^2)$

$\square - \square$

1) $\sqrt[3]{343x^3} = 7x$
2) $\sqrt[3]{8y^3} = 2y$
3) Subtr.

#12) $-1 - x^3$

$= -(x^3+1)$

$= -(x+1)(x^2 - x + 1)$

$\square + \square$

1) $\sqrt[3]{x^3} = x$
2) $\sqrt[3]{1} = 1$
3) Add'n

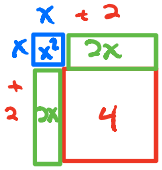
Factor each of the following polynomial functions whose equations and graphs are provided. Then, set each factor equal to zero and find the values of x.

$F(x) = 3x^3 + 12x^2 + 12x$

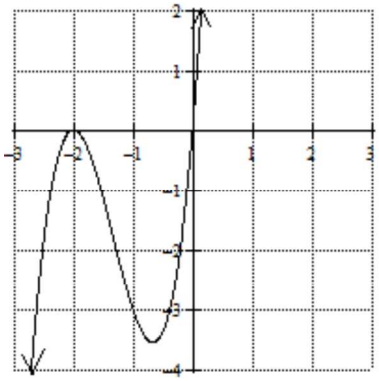
Perfect □ tri?

$0 = 3x(x^2 + 4x + 4)$

$0 = 3x(x+2)^2$



$0 = 3x$ $0 = x$ | $0 = x+2$ $-2 = x$ | $0 = x+2$ $-2 = x$



What is the y – intercept of the graph? 0

What is the constant term in the equation? None

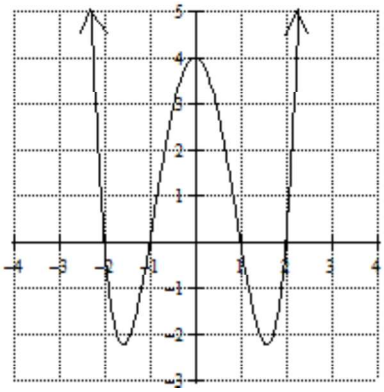
$G(x) = x^4 - 5x^2 + 4$

□-□? yes

$0 = (x^2-4)(x^2-1)$

$0 = (x-2)(x+2)(x-1)(x+1)$

$0 = x-2$ $2 = x$ | $x+2=0$ $x=-2$ | $x-1=0$ $x=1$ | $x+1=0$ $x=-1$



What is the y – intercept of the graph? 4

What is the constant term in the equation? 4

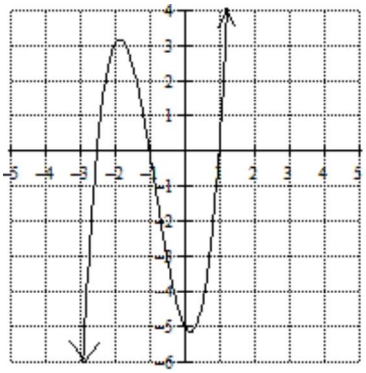
$H(x) = 2x^3 + 5x^2 - 2x - 5$

$0 = x^2(2x+5) - 1(2x+5)$

$0 = (2x+5)(x^2-1)$

$0 = (2x+5)(x-1)(x+1)$

$0 = 2x+5$ $-5 = 2x$ $-5/2 = x$ | $x-1=0$ $x=1$ | $x+1=0$ $x=-1$



What is the y – intercept of the graph? -5

What is the constant term in the equation? -5

$P(x) = 2x^4 - 5x^2 + 3$

$0 = 2x^4 - 3x^2 - 2x^2 + 3$

$M = 2x^4$
$A = -5x^2$
$N = -2x^2, -3x^2$

$0 = x^2(2x^2 - 3) - 1(2x^2 - 3)$

$0 = (2x^2 - 3)(x^2 - 1)$

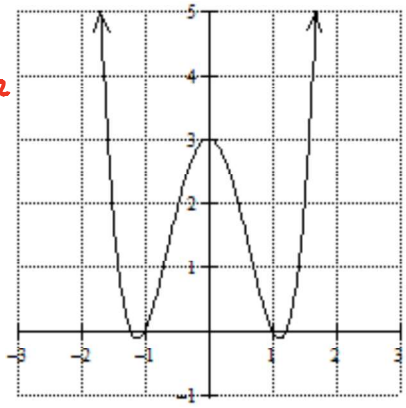
$0 = (2x^2 - 3)(x - 1)(x + 1)$

$0 = 2x^2 - 3$ | $0 = x - 1$ | $x + 1 = 0$

$3 = 2x^2$ | $1 = x$ | $x = -1$

$\frac{3}{2} = x^2$

$\pm\sqrt{\frac{3}{2}} = x$



What is the y – intercept of the graph? 3

What is the constant term in the equation? 3

What connection do you see between the equations of the polynomial and the graphs of the functions?

1. The constant term of the polynomial is the y-intercept
2. The degree of the function determines the max # of real zeros of the graph.
3. When the factors are set =0 and solved, the results are zeros of graph.