Notes 2.3 One – to – One Functions
Numerical, Graphical, and Analytical Approaches

A **function** is a relationship between x and y such that each x-value corresponds to only 1 y-value

If the point (2, –3) is on the graph of a function, $f(x)$, then no other point can have a(n) x-value of 2.

However, it is possible for another point on the graph of $f(x)$ to have a y – coordinate of –3. For example, both (2, –3) and (5, –3).

A **one-to-one function** is a function in which each y-value only has 1 x-value.

For example, if (2, –3) and (5, –3) are both on the graph of $f(x)$, then there is a y value, namely –3, that has more than one x value. Therefore, there is no way that the function is a one – to – one function.

Numerical Determination if a function is a one-to-one function.	• If f(x) is a continuous function, it is 1-1 if the function is always increasing or decreasing. • If f(x) is a discrete function, it is 1-1 if no y-values are repeated
Graphical Determination if a function is a one-to-one function.	If the graph of f(x) passes the HLT then it is 1-1
Analytical Determination if a function is a one-to-one function.	Based upon the equation and what type of function f(x) is, the shape of the graph should be known (or found out on calculator)

The **inverse** of a function is defined to be the function formed when the domain and range of a function are switched

What property of the function must exist in order for the inverse of the function to exist? Why?

f(x) must be 1-1 if $f^{-1}(x)$ is to exist.

B/c if f(x) is not 1-1, the repeated y-values of f(x) would become the repeated x-values of $f^{-1}(x)$.

If it can be determined that a function is one-to-one, then the inverse of the function can be found. Fill in the following chart which will describe how to find the inverse of the function if it is established to be a one-to-one function.

Numerical Determination of the inverse of a function	Switch the x and y-values
Graphical Determination of the inverse of a function	reflect the graph over the line y=x
Analytical Determination of the inverse of a function	• Switch the x and y in the equation • solve for y • replace y with inverse notation

Examples: For each of the following functions, determine if the inverse of the function exists, providing justification. If it does, then find the inverse. If the relation is given in numerical form, give the table of values that would represent $f^{-1}(x)$. If the relation is given in graphical form, sketch the graph of $f^{-1}(x)$. If the relation is given in analytical form, find the equation of $f^{-1}(x)$.

a.

x	–3	0	2	3	5
$F(x)$	2	–1	–3	0	2

The y-value 2 is paired with two different x-values

∴ F(x) is not 1-1

∴ $F^{-1}(x)$ does not exist

b.

x	–1	1	5	3	2
$F(x)$	6	–2	–1	2	3

Each y-value is paired with one x-value.

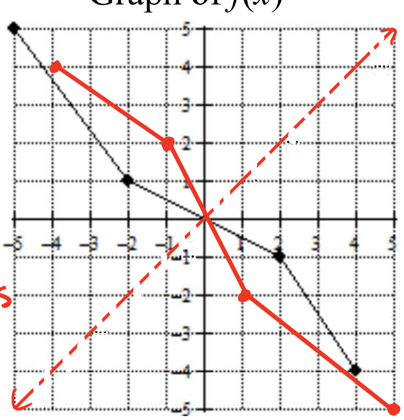
∴ F(x) is 1-1

∴ $F^{-1}(x)$ exists

x	6	-2	-1	2	3
$F^{-1}(x)$	-1	1	5	3	2

c. Graph of $f(x)$

F	F^{-1}
(-5,5)	(5,-5)
(-2,1)	(1,-2)
(2,-1)	(-1,2)
(4,-4)	(-4,4)

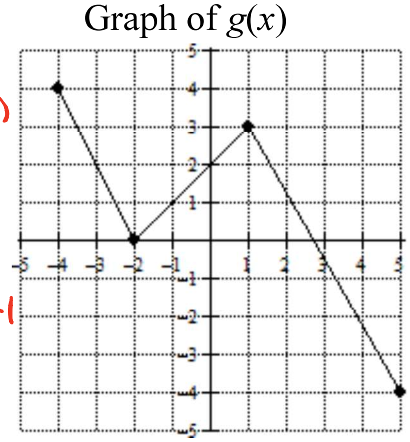


F(x) is always decreasing on its domain.

∴ F(x) is 1-1

∴ $F^{-1}(x)$ exists

d. Graph of $g(x)$



The graph of g(x) does not pass the HLT

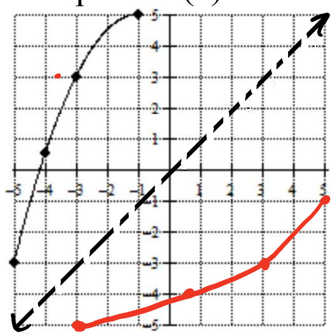
∴ g(x) is not 1-1

∴ $g^{-1}(x)$ does not exist

e.

$h(x)$	$h^{-1}(x)$
$(-5,-3)$	$(-3,-5)$
$(-4,1)$	$(1,-4)$
$(-3,3)$	$(3,-3)$
$(-1,5)$	$(5,-1)$

Graph of $h(x)$



The graph of $h(x)$ passes the HLT

$\therefore h(x)$ is 1-1

$\therefore h^{-1}(x)$ exists

f. $f(x) = (x-3)^2 - 2$

The graph of $f(x)$ is a parabola and wouldn't pass the HLT.

$\therefore f(x)$ is not 1-1

$\therefore f^{-1}$ doesn't exist

g. $f(x) = \frac{2}{5}x - 3$

The graph of $f(x)$ is a non-horizontal line and passes the HLT.

$\therefore f(x)$ is 1-1

$\therefore f^{-1}(x)$ exists

$x = \frac{2}{5}y - 3$

$x + 3 = \frac{2}{5}y$

$\frac{5}{2}x + \frac{15}{2} = y$

$f^{-1}(x) = \frac{5}{2}x + \frac{15}{2}$

h. $g(x) = 2\sqrt{x+3} + 4$

The graph of $g(x)$ is half a parabola and passes the HLT

$\therefore g(x)$ is 1-1

$\therefore g^{-1}(x)$ exists

$x = 2\sqrt{y+3} + 4$

$x - 4 = 2\sqrt{y+3}$

$\left(\frac{1}{2}(x-4)\right)^2 = \left(\sqrt{y+3}\right)^2$

$\frac{1}{4}(x-4)^2 = y+3$

$\frac{1}{4}(x-4)^2 - 3 = y$

$g^{-1}(x) = \frac{1}{4}(x-4)^2 - 3, \quad x \geq 4$

Ask why?

Let $g(x) = \dfrac{3x-4}{2}$.

Find $g^{-1}(x)$	Find $g(g^{-1}(x))$.	Find $g^{-1}(g(x))$.
$x = \dfrac{3y-4}{2}$ $2x = 3y-4$ $2x+4 = 3y$ $f^{-1}(x) = \dfrac{2x+4}{3}$	$g\left(\dfrac{2x+4}{3}\right) = \dfrac{3\left(\frac{2x+4}{3}\right)-4}{2}$ $= \dfrac{2x+4-4}{2}$ $= \dfrac{2x}{2}$ $= x$	$g^{-1}\left(\dfrac{3x-4}{2}\right) = \dfrac{2\left(\frac{3x-4}{2}\right)+4}{3}$ $= \dfrac{3x-4+4}{3}$ $= \dfrac{3x}{3}$ $= x$

If two functions, $f(x)$ and $g(x)$, are inverses of each other, then there are two special composite functions that will equal the same thing:

$\boxed{f(g(x)) = x}$ AND $\boxed{g(f(x)) = x}$

Examples: Determine if each of the pairs of functions below are inverse functions of each other or not. Show your work.

a. $f(x) = 2x + 3$ $\quad g(x) = \dfrac{x-3}{2}$

$f(g(x)) = 2\left(\frac{x-3}{2}\right)+3$

$= x-3+3$

$f(g(x)) = x$

$g(f(x)) = \dfrac{(2x+3)-3}{2}$

$= \dfrac{2x}{2}$

$g(f(x)) = x$

$\therefore f(g(x)) = g(f(x)) = x$

$\therefore$ $f(x)$ and $g(x)$ are inverses

b. $f(x) = \dfrac{x}{2} - 1$ $\quad g(x) = 2x + 1$

$f(g(x)) = \dfrac{(2x+1)}{2} - 1$

$= \frac{2x}{2} + \frac{1}{2} - 1$

$= x - \frac{1}{2}$

$\therefore f(g(x)) \neq x$

$\therefore$ f and g are not inverses