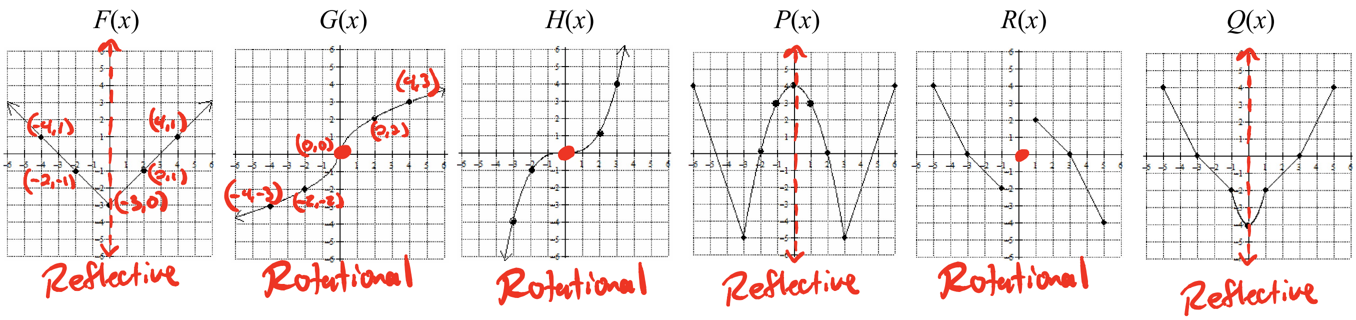
Notes 2.2 Even and Odd Functions

In geometry, you learned about two types of symmetry – reflective and rotational symmetry. Determine which of the following graphs exhibit reflective symmetry and which graphs exhibit rotational symmetry. For those that exhibit reflective symmetry, draw the line of symmetry using a dashed line. For those that exhibit rotational symmetry, identify the point of symmetry.



Above, there are three functions that exhibit reflective symmetry and their line of symmetry is the y – axis. Also, there are three functions that exhibit rotational symmetry and their point of symmetry is the origin.

In the boxes, identify the three functions that have reflective symmetry and the three functions that have rotational symmetry. Then, in the boxes below each function, list all of the specifically plotted ordered pairs on the graph. ~~List them in order that they appear from left to right on the graph.~~

Functions with Y – Axis Reflective Symmetry

F(x)	P(x)	Q(x)
(-4,1) (4,1) (-2,-1) (2,1) (-3,0)	(-6,4) (6,4) (-3,-5) (3,-5) (-2,0) (2,0) (-1,3) (1,3) (0,4)	(-5,4) (5,4) (-3,0) (3,0) (-1,2) (1,2) (0,-4)

Functions with Origin Rotational Symmetry

G(x)	H(x)	R(x)
(-4,-3) (4,3) (-2,-2) (2,2) (0,0)	(-3,-4) (3,4) (-2,-1) (2,1) (0,0)	(-5,4) (5,-4) (-3,0) (3,0) (-1,-2) (1,2)

What observation do you make about all of the points on the graphs of the functions with y – axis reflective symmetry?

$(x,y) \to (-x,y)$

What observation do you make about all of the points on the graphs of the functions with origin rotational symmetry?

$(x,y) \to (-x,-y)$

Numerical and Graphical Definition of Even and Odd Functions

Even Function	Numerically a function is even if for every point (x,y) there exists an $(-x,y)$ Graphically, a function is EVEN if it has reflectional symmetry with the y-axis
Odd Function	Numerically a function is ODD if for every point (x,y) there exists an $(-x,-y)$ Graphically, a function is ODD if it has rotational symmetry with the origin.

Analytically, a function is an even function if the equation of the function $f(-x)$ can be simplified to be the same equation as that of the function $f(x)$. Thus, $f(-x) = f(x)$.

Example: $f(x) = 2|3x| + x^2$

$f(-x) = 2|3(-x)| + (-x)^2$

$= 2|-3x| + x^2$

$f(-x) = 2|3x| + x^2$

$f(x) = f(-x)$

$\therefore f(x)$ is Even

Example: $g(x) = 2x^4 + x^2$

$g(-x) = 2(-x)^4 + (-x)^2$

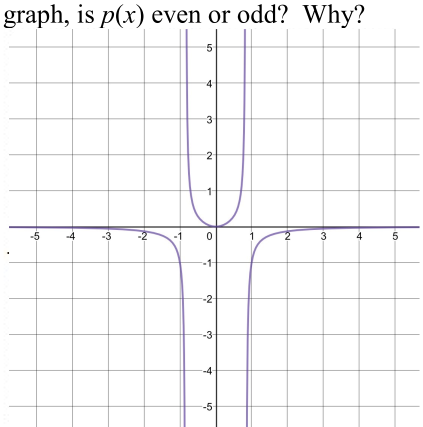
$g(-x) = 2x^4 + x^2$

$g(x) = g(-x)$

$\therefore g(x)$ is EVEN

Let's consider the function $p(x) = \frac{x^3}{x - 2x^5}$.

Using your graphing calculator, sketch the graph of $p(x)$ on the axes below. Based on the graph, is $p(x)$ even or odd? Why?



The graph has reflectional symmetry with the y-axis.

$\therefore p(x)$ is even

Find an equation for $p(-x)$. Based on the equation, is $p(x)$ the same type of function—even or odd—that you identified from the graph? Why or why not?

$p(-x) = \frac{(-x)^3}{(-x) - 2(-x)^5}$

$= \frac{-x^3}{-x - 2(-x^5)}$

$= \frac{-x^3}{-x + 2x^5}$

$= \frac{-1 \cdot x^3}{-1(x - 2x^5)}$

$p(-x) = \frac{x^3}{x - 2x^5}$

$p(x) = p(-x)$

$\therefore p(x)$ is Even

Analytically, a function is an odd function if the equation of the function $f(-x)$ can be simplified to be the NEGATIVE of the function $f(x)$. Thus, $f(-x) = -f(x)$.

Example: $f(x) = 3x + x^3$

$f(-x) = 3(-x) + (-x)^3$

$= -3x - x^3$

$f(-x) = -(3x + x^3)$

$f(-x) = -f(x)$

$\therefore f(x)$ is ODD

Example: $g(x) = 2x - x|2x|$

$g(-x) = 2(-x) - (-x)|2(-x)|$

$= -2x + x|-2x|$

$= -2x + x|2x|$

$g(-x) = -[2x - x|2x|]$

$g(-x) = -g(x)$

$\therefore g(x)$ is ODD

If $f(-x)$ does not simplify to the same equation as $f(x)$ or $-f(x)$, then the function is neither even nor odd.

Example $f(x) = 3x^2 + x$

$f(-x) = 3(-x)^2 + (-x)$

$= 3 \cdot x^2 - x$

$= 3x^2 - x$

$f(-x) = -(-3x^2 + x)$

$f(-x) \neq f(x)$

$f(-x) \neq -f(x)$

$\therefore f(x)$ is neither ODD or even

Example: $g(x) = 2x|x| + x^2$

$g(-x) = 2(-x)|-x| + (-x)^2$

$= -2x|x| + x^2$

$g(-x) = -[2x|x| - x^2]$

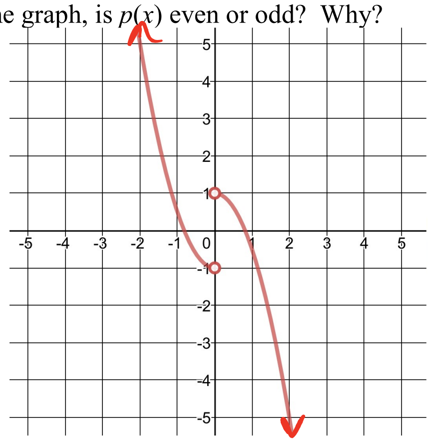
$f(-x) \neq f(x)$

$f(-x) \neq -f(x)$

$\therefore g(x)$ is neither ODD or even

Let's consider one last example, the function $p(x) = \frac{2x - 3x^3}{2|x|}$.

Using your graphing calculator, sketch the graph of $p(x)$ on the axes below. Based on the graph, is $p(x)$ even or odd? Why?



The graph has rotational symmetry with the origin

$\therefore p(x)$ is ODD

Find an equation for $p(-x)$. Based on the equation, is $p(x)$ the same type of function—even or odd—that you identified from the graph? Why or why not?

$p(-x) = \frac{2(-x) - 3(-x)^3}{2|-x|}$

$= \frac{-2x - 3(-x)^3}{2|x|}$

$= \frac{-2x + 3x^3}{2|x|}$

$p(-x) = -\frac{2x - 3x^3}{2|x|}$

$p(-x) = -p(x)$

$\therefore p(x)$ is ODD