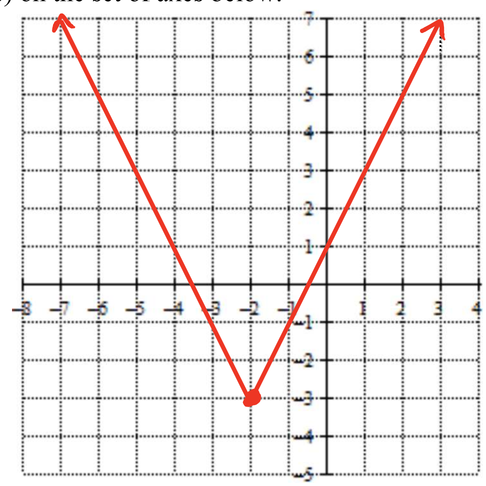
Notes 2.1 Rewriting Absolute Value Functions as Piece-wise Defined Functions

Consider the absolute value function $f(x) = 2|x+2| - 3$. Sketch the graph of $f(x)$ on the set of axes below.

vertex $(-2,-3)$



Now, write the graphed function as a two-piece, piece-wise defined function.

Equation of Each Piece		Constraint of Each Piece
$-2x-7$	,	$x \le -2$
$2x+1$	,	$x > -2$

$$f(x)=\begin{cases} -2x-7, & x \le -2 \\ 2x+1, & x > -2 \end{cases}$$

Follow the strategy outlined below for the function $f(x) = 2|x+2| - 3$.

1. Set each expression inside of absolute value = 0 and solve for *x*.	$x+2=0$ $x=-2$
2. Draw a number line and divide it into intervals using the values of *x* found in the previous step and write the entire equation of the function in each interval, replacing the absolute value bars with parenthesis.	$x=-3$ | $x=0$ -2 $y=-2(x+2)-3$ | $y=+2(x+2)-3$ $y=-2x-4-3$ | $y=2x+4-3$ $y=-2x-7$ | $y=2x+1$
3. Determine the sign of each expression in parenthesis by choosing a value from each interval and substituting in for *x*. Place the sign in front of the expressions on each interval.	Don't need to show $x=-3 \Rightarrow (-3+2)=-1$ (NEG) $x=0 \Rightarrow (0+2)=2$ (POS)
4. Simplify the equations on each interval by distributing the signs determined in the previous step and combining like terms. Then, write the equation as a piece-wise defined function.	$f(x)=\begin{cases} -2x-7 & x \le -2 \\ 2x+1 & x > -2 \end{cases}$

Compare your piece-wise defined function that you wrote in step #4 above to the equation that you wrote from the graph at the beginning of this exercise. Are they the same? Should they be the same? Explain.

They should be the same equation because both equations represent the same absolute value function, f(x)

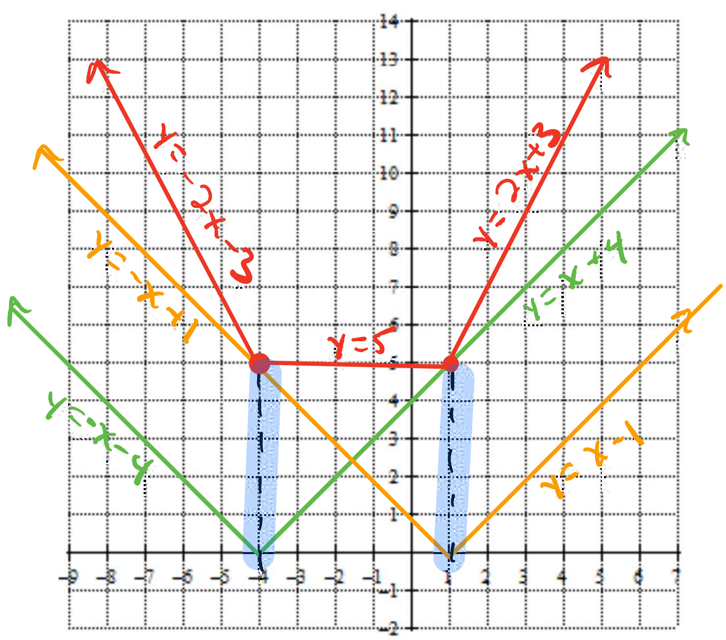
Follow the strategy outlined below for the function $g(x) = |x+4| + |x-1|$.

1. Set each expression inside of absolute value = 0 and solve for *x*.	$x+4=0$ $x=-4$ $x-1=0$ $x=1$
2. Draw a number line and divide it into intervals using the values of *x* found in the previous step and write the entire equation of the function in each interval, replacing the absolute value bars with parenthesis.	$x=-10$ -4 $x=0$ 1 $x=10$ $y=-(x+4)-(x-1)$ $y=+(x+4)-(x-1)$ $y=+(x+4)+(x-1)$
3. Determine the sign of each expression in parenthesis by choosing a value from each interval and substituting in for *x*. Place the sign in front of the expressions on each interval.	$y=-x-4-x+1$ $y=-2x-3$ $y=x+4-x+1$ $y=5$ $y=x+4+x-1$ $y=2x+3$
4. Simplify the equations on each interval by distributing the signs determined in the previous step and combining like terms. Then, write the equation as a piece-wise defined function.	$g(x)=\begin{cases} -2x-3, & x<-4 \\ 5, & -4\le x\le 1 \\ 2x+3, & x>1 \end{cases}$

Now, graph the function $g(x)$ on the grid below using the piece-wise defined function and check your graph in the graphing calculator. Then, identify the domain and range of $g(x)$.

Domain: $(-\infty,\infty)$

Range: $[5,\infty)$

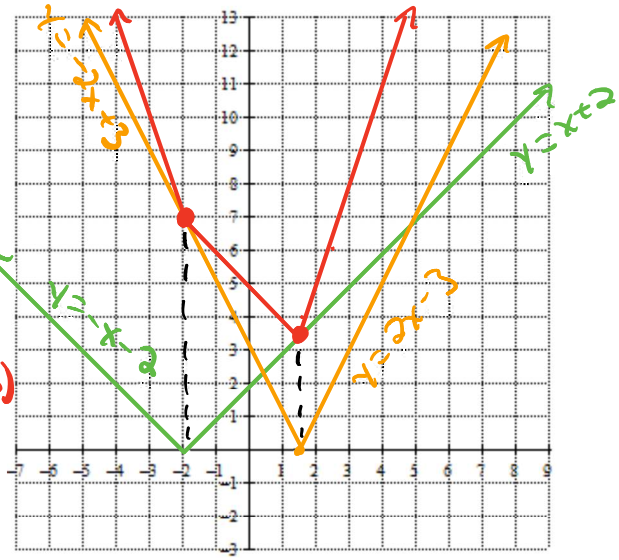


$|2(x-\tfrac{3}{2})|$

Using the strategy defined above, rewrite the function $f(x) = |x+2| + |2x-3|$ as a piece-wise defined function. Then, graph the function on the grid provided.

$x+2=0$
$x=-2$

$2x-3=0$
$2x=3$
$x=3/2$



$x=-10$ | $x=0$ | $x=10$

-2 | $3/2$

$y = -(x+2) + -(2x-3)$
$y = -x-2-2x+3$
$y = -3x+1$

$y = +(x+2) + -(2x-3)$
$y = x+2-2x+3$
$y = -x+5$

$y = +(x+2) + +(2x-3)$
$y = x+2+2x-3$
$y = 3x-1$

$$f(x) = \begin{cases} -3x+1, & x < -2 \\ -x+5, & -2 \le x \le 3/2 \\ 3x-1, & x > 3/2 \end{cases}$$

Perform the described process for the function $p(x) = \frac{|5-x|-8}{x+3}$.

$5-x=0$
$5=x$

Denom $\neq 0$
$x+3 \neq 0$
$x \neq -3$

$x=-1$ | $x=0$ | $x=10$

-3 | 5

$y = \frac{+(5-x)-8}{x+3}$
$y = \frac{5-x-8}{x+3}$
$y = \frac{-x-3}{x+3}$
$y = \frac{-(x+3)}{x+3}$
$y = -1$

$y = \frac{+(5-x)-8}{x+3}$
$y = -1$

$y = \frac{-(5-x)-8}{x+3}$
$y = \frac{-5+x-8}{x+3}$
$y = \frac{x-13}{x+3}$

$$p(x) = \begin{cases} -1, & x \le 5, x \neq -3 \\ \frac{x-13}{x+3}, & x > 5 \end{cases}$$

What must be taken into consideration for the function $p(x)$ that did not have to be considered before?

Since p(x) is rational, then the restricted values must be excluded from the constraints of the piece-wise function.