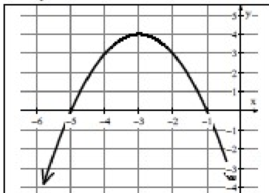
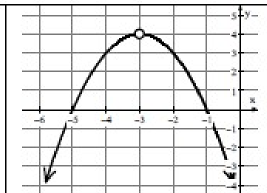
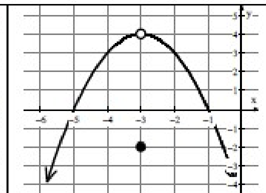
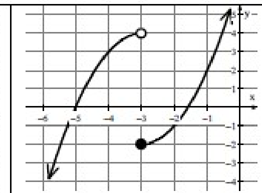
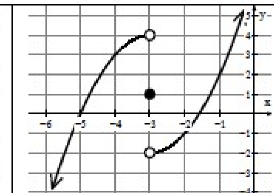
Notes 1.7 Discontinuities in the Graphs of Piece-wise Defined Functions

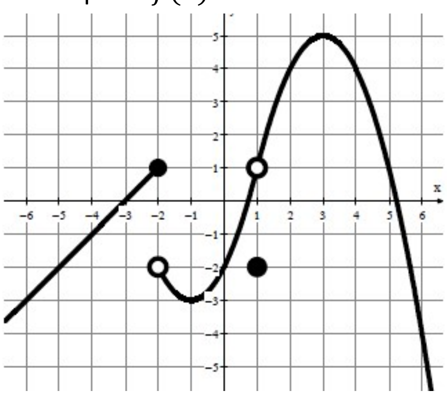
Limits

A ***one-sided limit*** is the <u>y-value</u> that a function approaches from either the <u>left</u> or the <u>right</u> side of a given <u>x-value</u>.

A ***limit*** is the <u>y-value</u> that a function <u>approaches</u> at a given <u>x-value</u> if the left-sided limit is equal to the right-sided limit.

				
$\lim_{x\to-3} f(x) =$ 4	$\lim_{x\to-3} f(x) =$ 4	$\lim_{x\to-3} f(x) =$ 4	$\lim_{x\to-3} f(x) =$ DNE	$\lim_{x\to-3} f(x) =$ DNE
$f(-3) =$ 4	$f(-3) =$ DNE	$f(-3) =$ -2	$f(-3) =$ -2	$f(-3) =$ 1

1. Graph of $f(x)$



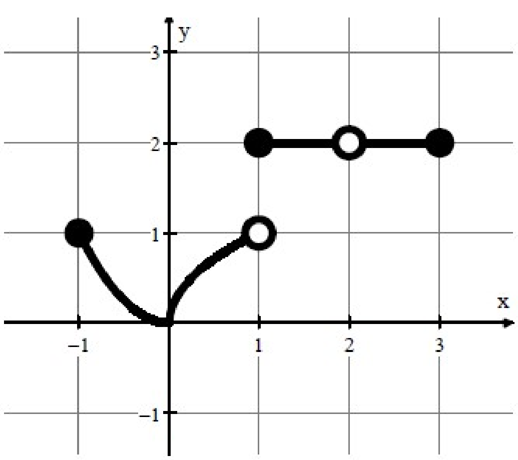
a. $\lim_{x\to-2^-} f(x) =$ 1 $\lim_{x\to-2^+} f(x) =$ -2 $\lim_{x\to-2} f(x) =$ DNE $f(-2) =$ 1

b. $\lim_{x\to0^-} f(x) =$ -2 $\lim_{x\to0^+} f(x) =$ -2 $\lim_{x\to0} f(x) =$ -2 $f(0) =$ -2

c. $\lim_{x\to1^-} f(x) =$ 1 $\lim_{x\to1^+} f(x) =$ 1 $\lim_{x\to1} f(x) =$ 1 $f(1) =$ -2

d. $\lim_{x\to3^-} f(x) =$ 5 $\lim_{x\to3^+} f(x) =$ 5 $\lim_{x\to3} f(x) =$ 5 $f(3) =$ 5

2. Graph of $g(x)$



Decide if each statement is True of False

$\lim_{x\to-1} g(x) = 1$	False	$\lim_{x\to2} g(x) = 2$	True
$\lim_{x\to0} g(x) = 0$	True	$\lim_{x\to1} g(x) = 1$	False
$\lim_{x\to1} g(x) = 2$	False	$\lim_{x\to1} g(x) = DNE$	True
$\lim_{x\to3} g(x) = 2$	False	$\lim_{x\to3} g(x) = DNE$	True

Continuous at $x = a$

$f(x)$ is continuous at $x = a$ if following are true

I. $f(a)$ is defined

II. $\lim_{x \to a} f(x)$ exists

III. $\lim_{x \to a} f(x) = f(a)$

Infinite Discontinuity at $x = a$

$f(x)$ has infinite discontinuity at $x = a$ if following are true

I. $f(a)$ is undefined

II. $\lim_{x \to a^-} f(x) = \pm\infty$

or

$\lim_{x \to a^+} f(x) = \pm\infty$

Jump Discontinuity at $x = a$

$f(x)$ has jump discontinuity at $x = a$ if following are true

I. $f(a)$ is or isn't defined

II. $\lim_{x \to a} f(x)$ doesn't exist

but $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist.

Point Discontinuity at $x = a$

$f(x)$ point discontinuity at $x = a$ if following are true

I. $f(a)$ is or isn't defined

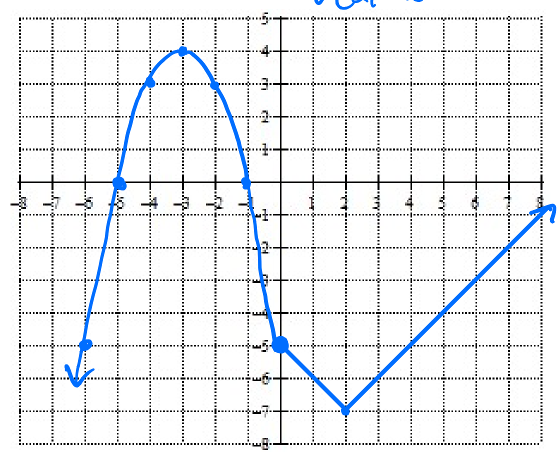
II. $\lim_{x \to a} f(x)$ exists

III. $\lim_{x \to a} f(x) \neq f(a)$

$v(-3,4)$

$$q(x) = \begin{cases} -(x+3)^2 + 4, & -5 \le x < 0 \\ |x-2| - 7, & x \ge 0 \end{cases}$$

$v(2,-7)$



1. Draw the graph of $q(x)$ on the grid.

2. Looking at the graph, determine if q(x) is continuous, has infinite discontinuity, jump discontinuity or point discontinuity at $x = 0$. Justify your answer.

I. $q(0) = -5$

$\therefore q(0)$ is defined

II. $\lim_{x \to 0^-} q(x) = \lim_{x \to 0^+} q(x) = -5$

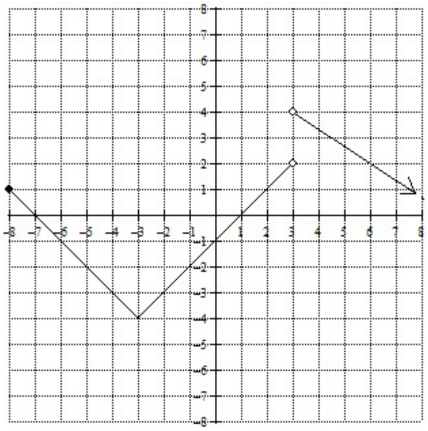
$\therefore \lim_{x \to 0} q(x)$ exists

III. $\lim_{x \to 0} q(x) = q(0) = -5$

$\therefore q(x)$ is continuous at $x = 0$

Discontinuities (Jump, Point & Infinite)

$$f(x)=\begin{cases}|x+3|-4, & -8\le x<3\\ -\dfrac{1}{3}x+5, & x>3\end{cases}$$



3. Looking at the graph, determine if f(x) is continuous, has infinite discontinuity, jump discontinuity or point discontinuity at $x=3$. Justify your answer.

I. $f(3)$ is undefined

II. $\lim\limits_{x\to 3^-} f(x) = 2$

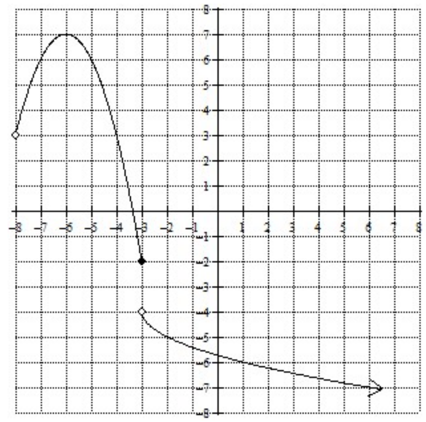
$\lim\limits_{x\to 3^+} f(x) = 4$

$\therefore \lim\limits_{x\to 3} f(x)$ does not exist

III $\lim\limits_{x\to 3} f(x) \neq f(3)$

$\therefore f(x)$ has jump discontinuity at $x=3$

$$g(x)=\begin{cases}-(x+6)^2+7, & -8<x\le -3\\ -\sqrt{x+3}-4, & x>-3\end{cases}$$



4. Looking at the graph, determine if g(x) is continuous, has infinite discontinuity, jump discontinuity or point discontinuity at $x=-3$. Justify your answer.

I. $g(-3) = -2$

$\therefore g(-3)$ is defined

II $\lim\limits_{x\to -3^-} g(x) = -2$

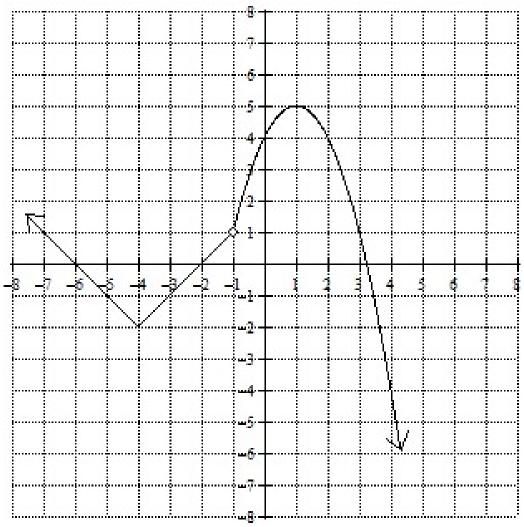
$\lim\limits_{x\to -3^+} g(x) = -4$

$\therefore \lim\limits_{x\to -3} g(x)$ does not exist

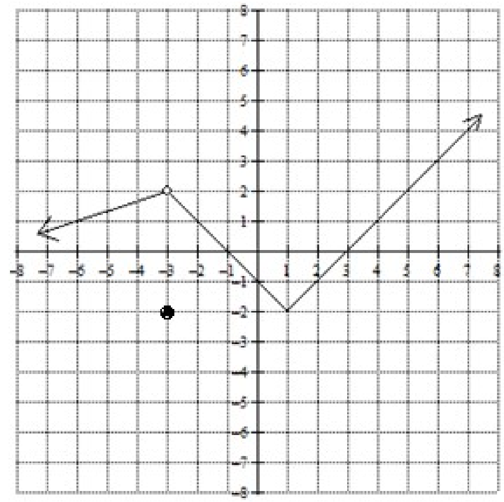
III $\lim\limits_{x\to -3} g(x) \neq g(-3)$

$\therefore g(x)$ has jump discontinuity at $x=-3$

$$h(x) = \begin{cases} |x+4| - 2, & x < -1 \\ -(x-1)^2 + 5, & x > -1 \end{cases}$$



$$p(x) = \begin{cases} \frac{1}{3}x + 3, & x < -3 \\ -2, & x = -3 \\ |x-1| - 2, & x > -3 \end{cases}$$



5. Looking at the graph, determine if h(x) is continuous, has infinite discontinuity, jump discontinuity or point discontinuity at $x = -1$. Justify your answer.

I. $h(-1)$ is not defined

II. $\lim_{x \to -1^-} h(x) = \lim_{x \to -1^+} h(x) = 1$

$\therefore \lim_{x \to -1} h(x)$ exists

III $\lim_{x \to -1} h(x) \neq h(-1)$

$\therefore$ h(x) has point discontinuity at $x = -1$

6. Looking at the graph, determine if p(x) is continuous, has infinite discontinuity, jump discontinuity or point discontinuity at $x = -3$. Justify your answer.

I. $p(-3) = -2$

$\therefore p(-3)$ is defined

II. $\lim_{x \to -3^-} p(x) = \lim_{x \to -3^+} p(x) = 2$

$\therefore \lim_{x \to -3} p(x)$ exists

III $\lim_{x \to -3} p(x) \neq p(-3)$

$\therefore$ p(x) has point discontinuity at $X = -3$

7. Does $g(x)$ have a discontinuity at $x = -1$? If so, classify it and justify your reasoning.

$$g(x) = \begin{cases} -(x+4)^2 + 3, & x < -1 \\ |x-3| - 10, & x > -1 \end{cases}$$

I. $g(-1)$ is not defined

II. $\lim_{x \to -1^-} g(x) = -(-1+4)^2 + 3 = -(3)^2 + 3 = -9 + 3 = -6$

$\lim_{x \to -1^+} g(x) = |-1-3| - 10 = |-4| - 10 = 4 - 10 = -6$

$\lim_{x \to -1^-} g(x) = \lim_{x \to -1^+} g(x) = -6$

$\therefore \lim_{x \to -1} g(x)$ exists.

III $\lim_{x \to -1} g(x) \neq g(-1)$

$\therefore$ $g(x)$ has point discontinuity at $x = -1$

8. Does $g(x)$ have a discontinuity at $x = -1$? If so, classify it and justify your reasoning.

$$g(x) = \begin{cases} -2|x+3| + 2, & -6 < x < -1 \\ |x+3| - 4, & x \geq -1 \end{cases}$$

9. Does $h(x)$ have a discontinuity at $x = 3$ or $x = 5$? If so, classify it and justify your reasoning.

$$h(x) = \begin{cases} 2x - 3, & x < 3 \\ x^2 - 2x, & 3 \leq x < 5 \\ |x-3| + 2, & x \geq 5 \end{cases}$$

I. $h(3) = (3)^2 - 2(3) = 9 - 6 = 3$

$\therefore$ $h(3)$ is defined

II $\lim_{x \to 3^-} h(x) = 2(3) - 3 = 6 - 3 = 3$

$\lim_{x \to 3^+} h(x) = (3)^2 - 2(3) = 9 - 6 = 3$

$\lim_{x \to 3^-} h(x) = \lim_{x \to 3^+} h(x)$

$\therefore \lim_{x \to 3} h(x)$ exists

III $\lim_{x \to 3} h(x) = h(3)$

$\therefore$ $h(x)$ is continuous at $x = 3$

I. $h(5) = |5-3| + 2 = |2| + 2 = 2 + 2 = 4$

$\therefore$ $h(5)$ is defined

II $\lim_{x \to 5^-} h(x) = (5)^2 - 2(5) = 25 - 10 = 15$

$\lim_{x \to 5^+} h(x) = |5-3| + 2 = |2| + 2 = 2 + 2 = 4$

$\lim_{x \to 5^-} h(x) \neq \lim_{x \to 5^+} h(x)$

$\therefore \lim_{x \to 5} h(x)$ does not exist

$\therefore$ $h(x)$ has jump discontinuity at $x = 5$.

10. For what value(s) of *a* and *b* will the function below have a point discontinuities at *x* = −3 and *x* = 2?

$$f(x) = \begin{cases} 2x + 3, & x < -3 \\ ax + 3b, & -3 < x < 2 \\ \frac{1}{2}x^2 + 3x - 1, & x > 2 \end{cases}$$

Point Disc @ x = -3

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x)$

$2(-3) + 3 = a(3) + 3b$

$-6 + 3 = 3a + 3b$

$-3 = 3a + 3b$

times(-2) $-1 = a + b$

Point Disc @ x = 2

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$a(2) + 3b = \frac{1}{2}(2)^2 + 3(2) - 1$

$2a + 3b = \frac{1}{2}(4) + 6 - 1$

$2a + 3b = 2 + 5$

$2a + 3b = 7$

System of equations

$$\begin{array}{r} 2a + 3b = 7 \\ -2a - 2b = 2 \\ \hline b = 9 \end{array}$$

$a + b = -1$

$a + (9) = -1$

$a = -10$

f(x) will have point discontinuities at x = -3 and x = 2, if a = -10 and b = 9