

**Notes 1.6 Composition of Functions**  
**Numerical, Graphical, and Analytical Approaches**

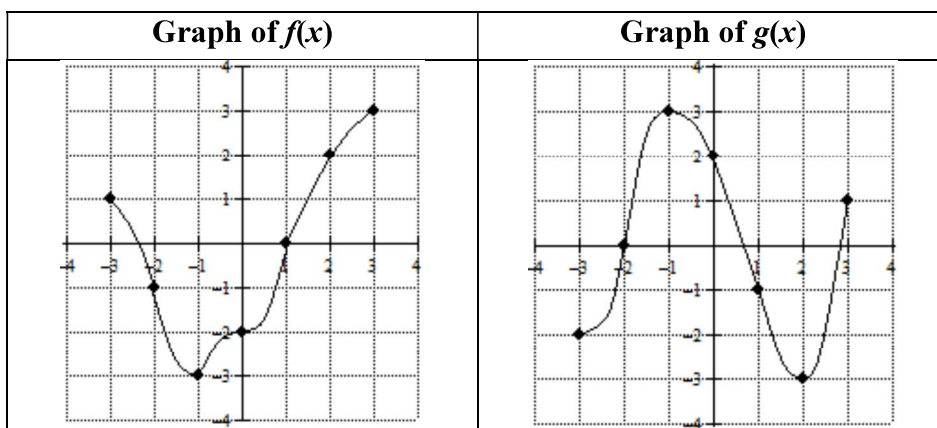
Given below is a table of values for three functions,  $f(x)$ ,  $g(x)$ , and  $h(x)$ . Find each of the following composite function values based on the table values.

$x$	-3	-2	-1	0	1	2	3
$f(x)$	0	3	2	-4	4	3	2
$g(x)$	-2	1	7	-2	0	5	-3
$h(x)$	$\frac{1}{2}$	-1	1	2	-3	0	-1

SHOW EACH STEP OF YOUR ANALYSIS!!!!!!

a) $-3h(0) \cdot g(-3)$ $= -3(2) \cdot (-2)$ $= -6(-2)$ $= 12$	b) $\frac{2g(-1)}{f(-2)} = \frac{2(7)}{3}$ $= \frac{14}{3}$	c) $h(-2) + 3g(-1) = (-1) + 3(7)$ $= -1 + 21$ $= 20$
d) $(h \cdot f)(2) = h(2) \cdot f(2)$ $= 0 \cdot 3$ $= 0$	e) $h(f(2)) = h(3)$ $= -1$	f) $g(f(-3)) = g(0)$ $= -2$
g) $h(g(-2)) = h(1)$ $= -3$	h) $g(f(3)) = g(2)$ $= 5$	i) $f(h(1)) = f(-3)$ $= 0$
j) $h(g(f(2))) = h(g(3))$ $= h(-3)$ $= \frac{1}{2}$	k) $g(h(f(-1))) = g(h(2))$ $= g(0)$ $= -2$	l) $g(f(g(-3))) = g(f(-2))$ $= g(3)$ $= -3$
m) If $k(x) = 2x + 10$ , then for what value(s) of $x$ is $k(x) = f(g(h(-1)))$ . $2x + 10 = f(g(1))$ $= f(0)$ $2x + 10 = -4$ $2x = -14$ $x = -7$	n) If $p(x) =  2x - 3 $ , then for what value(s) of $x$ is $p(x) = f(g(-2))$ ? $ 2x - 3  = f(1)$ $ 2x - 3  = 4$ $2x - 3 = \pm 4$ $2x = 3 \pm 4$ $x = \frac{3 \pm 4}{2}$ $x = -\frac{1}{2}, \frac{7}{2}$	

Pictured below are two functions,  $f(x)$  and  $g(x)$ . Use the graphs to find the indicated quantities.



SHOW EACH STEP OF YOUR ANALYSIS!!!!!!

<p>a) <math>-3f(2) + g(-3) =</math>  <math>-3(2) + (-2)</math>  <math>= -6 - 2</math>  <math>= -8</math></p>	<p>b) <math>g(3) - f(-1) = 1 - (-3)</math>  <math>= 4</math></p>	<p>c) <math>g(-2) \cdot f(3) = 0 \cdot 3</math>  <math>= 0</math></p>
<p>d) <math>g(f(-1)) = g(-3)</math>  <math>= -2</math></p>	<p>e) <math>f(g(0)) = f(2)</math>  <math>= 2</math></p>	<p>f) <math>g(f(-2)) = g(-1)</math>  <math>= 3</math></p>
<p>g) <math>g(f(0)) = g(-2)</math>  <math>= 0</math></p>	<p>h) <math>f(g(-3)) = f(-2)</math>  <math>= -1</math></p>	<p>i) <math>f(g(-1)) = f(3)</math>  <math>= 3</math></p>
<p>j) <math>g(f(3)) = g(3)</math>  <math>= 1</math></p>	<p>k) <math>f(g(f(1))) = f(g(0))</math>  <math>= f(2)</math>  <math>= 2</math></p>	<p>l) <math>g(g(f(-3))) = g(g(1))</math>  <math>= g(-1)</math>  <math>= 3</math></p>
<p>m) If <math>h(x) = x^2 - x</math>, then for what value(s) of <math>x</math> is <math>h(x) = g(f(1))</math>?</p> <p style="margin-left: 20px;"> <math>x^2 - x = g(0)</math>  <math>x^2 - x = 2</math>  <math>x^2 - x - 2 = 0</math>  <math>(x-2)(x+1) = 0</math>  <math>x-2=0 \quad \left\{ \begin{array}{l} x+1=0 \\ x=2 \quad \left\{ \begin{array}{l} x=-1 \end{array} \right. \end{array} \right.</math> </p>	<p>n) If <math>p(x) = 3 x - 2 </math>, then for what value(s) of <math>x</math> is <math>p(x) = f(g(-2))</math>?</p> <p style="margin-left: 20px;"> <math>3 x-2  = f(0)</math>  <math>3 x-2  = -2</math>  <math> x-2  = -2/3</math>  <b>NO SOLUTION</b> </p>	

**Using Equations to Find Numerical Compositions**

Find each of the indicated functions or quantities below using the four functions  $f$ ,  $g$ ,  $h$ , and  $p$ .

$f(x) = -x^2 + 3x - 5$	$g(x) = 1 - 2x$	$h(x) = \frac{3}{2}x - 2$	$p(x) = \frac{x+2}{x-3}$
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<p>1) <math>g(h(4)) = -7</math></p> $h(4) = \frac{3}{2}(4) - 2$ $= 6 - 2$ $h(4) = 4$ <hr style="border: 1px solid red;"/> $g(4) = 1 - 2(4)$ $= 1 - 8$ $= -7$	<p>2) <math>f(-2) + p(2) = -15 + -4</math></p> $= -19$ <hr style="border: 1px solid red;"/> $f(-2) = -(-2)^2 + 3(-2) - 5$ $= -4 - 6 - 5$ $= -15$ <hr style="border: 1px solid red;"/> $p(2) = \frac{(2)+2}{(2)-3} = \frac{4}{-1} = -4$	<p>3) <math>p(g(-1)) = \text{undefined}</math></p> <hr style="border: 1px solid red;"/> $g(-1) = 1 - 2(-1)$ $= 1 + 2$ $= 3$ <hr style="border: 1px solid red;"/> $p(3) = \frac{(3)+2}{(3)-3} = \frac{5}{0}$ $= \text{undefined}$
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**Using Equations to Find Algebraic Compositions**

Suppose that  $f(x) = 2x^2 - x + 3$  and  $g(x) = 5 - 2(x - 3)$ .

<p>Find an expression for <math>f(x + 3)</math>.</p> $f(x+3) = 2(x+3)^2 - (x+3) + 3$ $= 2(x^2 + 6x + 9) - x - 3 + 3$ $= 2x^2 + 12x + 18 - x$ $= 2x^2 + 11x + 18$	<p>Find an expression for <math>f(x - 2) - g(2x)</math>.</p> $f(x-2) = [2(x-2)^2 - (x-2) + 3] - [5 - 2(6x-3)]$ $= 2(x^2 - 4x + 4) - x + 2 + 3 - [5 - 4x + 6]$ $= 2x^2 - 8x + 8 - x + 5 - 5 + 4x - 6$ $= 2x^2 - 5x + 2$
<p>Find an expression for <math>\frac{g(x+h) - g(x)}{h}</math>.</p> $= \frac{[5 - 2((x+h) - 3)] - [5 - 2(x - 3)]}{h}$ $= \frac{[5 - 2x - 2h + 6] - [5 - 2x + 6]}{h}$ $= \frac{11 - 2x - 2h + 2x - 11}{h}$ $= \frac{-2h}{h}$ $= -2$	<p>Find an expression for <math>\frac{f(x+h) - f(x)}{h}</math>.</p> $= \frac{[2(x+h)^2 - (x+h) + 3] - [2x^2 - x + 3]}{h}$ $= \frac{2(x^2 + 2hx + h^2) - x - h + 3 - 2x^2 + x - 3}{h}$ $= \frac{2x^2 + 4hx + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h}$ $= \frac{4hx + 2h^2 - h}{h}$ $= \frac{h(4x + 2h - 1)}{h} = 4x + 2h - 1$

$$f(x) = -x^2 + 3x - 5 \quad g(x) = 1 - 2x \quad h(x) = \frac{3}{2}x - 2 \quad p(x) = \frac{x+2}{x-3}$$

<p>a) Find an equation for <math>g(h(x))</math>.</p> $= g\left(\frac{3}{2}x - 2\right) = 1 - 2\left(\frac{3}{2}x - 2\right)$ $= 1 - 3x + 4$ $= 5 - 3x$ <p><i>Expression</i></p>	<p>b) Find an equation for <math>g(f(x))</math>.</p> $g(-x^2 + 3x - 5) = 1 - 2(-x^2 + 3x - 5)$ $= 1 + 2x^2 - 6x + 10$ $= 2x^2 - 6x + 11$
<p>c) Find an <del>equation</del> for <math>f(g(x))</math>.</p> $f(1 - 2x) = -(1 - 2x)^2 + 3(1 - 2x) - 5$ $= -(1 - 4x + 4x^2) + 3 - 6x - 5$ $= -1 + 4x - 4x^2 + 3 - 6x - 5$ $= -4x^2 - 2x - 3$ <p><i>Expression</i></p>	<p>d) Find an equation for <math>p(g(x))</math>.</p> $p(1 - 2x) = \frac{(1 - 2x) + 2}{(1 - 2x) - 3}$ $= \frac{3 - 2x}{-2 - 2x}$
<p>e) Find an <del>equation</del> for <math>[-6x \cdot h(x) - f(x)]</math>.</p> $= -6x\left(\frac{3}{2}x - 2\right) - (-x^2 + 3x - 5)$ $= -9x^2 + 12x + x^2 - 3x + 5$ $= -8x^2 + 9x + 5$ <p><i>Expression</i></p>	<p>f) For what value(s) of <math>x</math> is <math>f(x) = g(x)</math>?</p> $-x^2 + 3x - 5 = 1 - 2x$ $0 = x^2 - 5x + 6$ $0 = (x - 3)(x - 2)$ $0 = x - 3 \quad \left. \begin{array}{l} 0 = x - 2 \\ 3 = x \end{array} \right\} 2 = x$ $x = 2, 3$
<p>g) Find an <del>equation</del> for <math>(f \cdot g)(x)</math>.</p> $= (-x^2 + 3x - 5)(1 - 2x)$ $= -x^2 + 3x - 5 + 2x^3 - 6x^2 + 10x$ $= 2x^3 - 7x^2 + 13x - 5$ <p><i>Expression</i></p>	
<p>h) Find an <del>equation</del> for <math>(g + p)(x)</math>.</p> $= g(x) + p(x)$ $= (1 - 2x) + \frac{x+2}{x-3} = \frac{(1-2x)(x-3)}{x-3} + \frac{x+2}{x-3}$ $= \frac{1-2x-2x^2-6x}{x-3} + \frac{x+2}{x-3} = \frac{-2x^2-8x+3}{x-3}$	