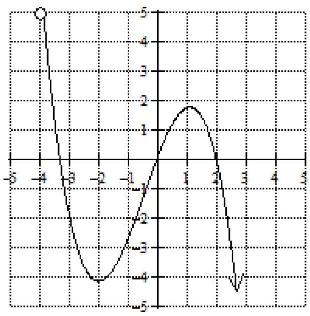
Notes 1.5 Determining Domain and Range of a Function

Define **domain**: All values of x that have a y-value

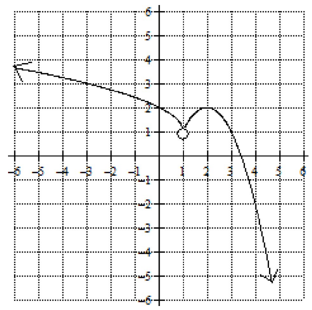
Define **range**: All values of y that have a x-value

Using interval notation, identify the domain and range of each graphed function below.

1.


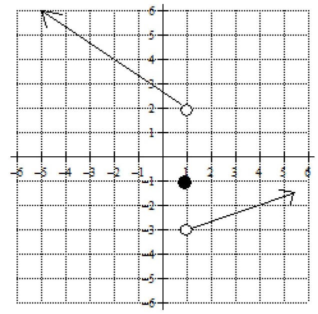
Domain: $(-4, \infty)$

Range: $(-\infty, 5)$

2.


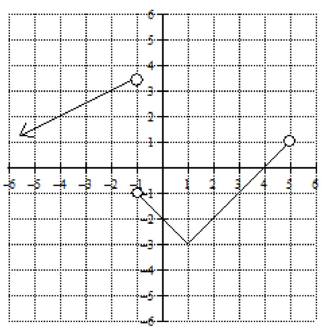
Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, \infty)$

3.


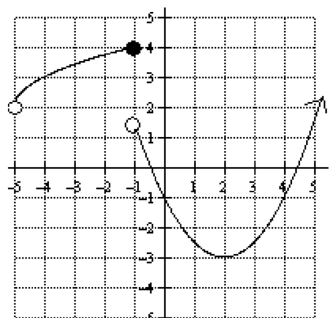
Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

4.


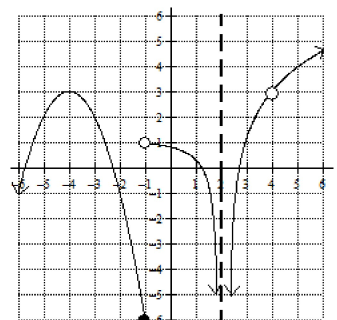
Domain: $(-\infty, -1) \cup (-1, 5)$

Range: $(-\infty, 3.5)$

5.


Domain: $(-5, \infty)$

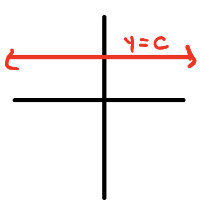
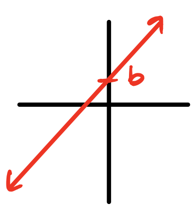
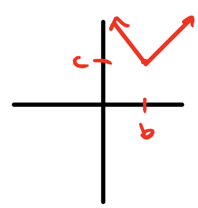
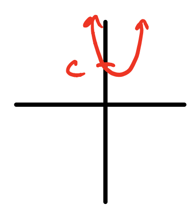
Range: $[-3, \infty)$

6.


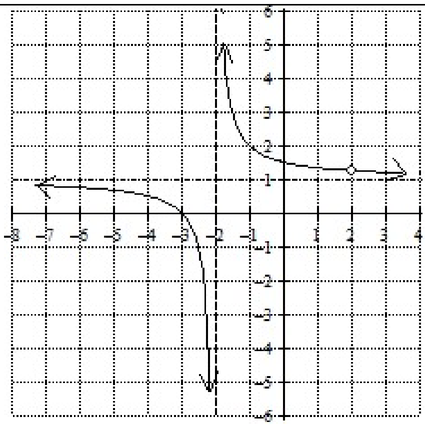
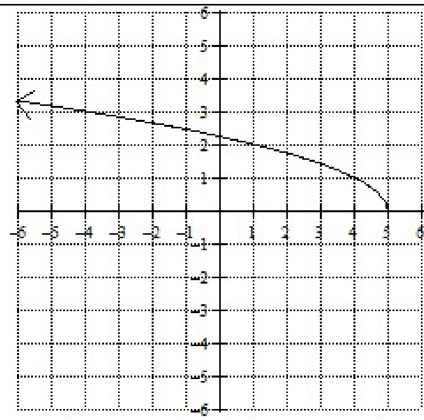
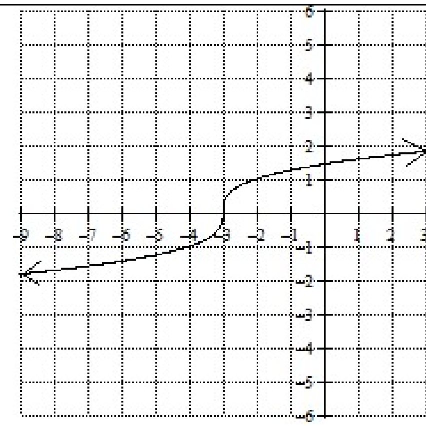
Domain: $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

Range: $(-\infty, \infty)$

Determining Domain of a Function Analytically

Constant Functions $f(x) = c$	**Linear Functions** $f(x) = ax + b$	**Absolute Value Functions** $f(x) = a\lvert x - b\rvert + c$	**Quadratic Functions** $f(x) = ax^2 + bx + c$
Sketch a general graph. 	Sketch a general graph. 	Sketch a general graph. 	Sketch a general graph. 
What is the domain? $(-\infty, \infty)$	What is the domain? $(-\infty, \infty)$	What is the domain? $(-\infty, \infty)$	What is the domain? $(-\infty, \infty)$

Given below are the graphs of three different functions that we will investigate analytically. Refer to them as you complete the table below. Identify the domain of each function graphed below.

Name: RATIONAL	Name: SQUARE ROOT	Name: Cube Root
$f(x)=\dfrac{x^2+x-6}{x^2-4}$	$g(x)=\sqrt{5-x}$	$h(x)=\sqrt[3]{x+3}$
		
Domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$	**Domain:** $(-\infty,5]$	**Domain:** $(-\infty,\infty)$

Complete the table below as a precursor to determining the domain of each function analytically.

Function	Find the indicated function value.	Find the indicated function value.	What conclusions can you make about the domain of the function based on these two computations?
$f(x)=\dfrac{x^2+x-6}{x^2-4}$	$f(-2)=\dfrac{(-2)^2+(-2)-6}{(-2)^2-4}$ $=\dfrac{4-8}{4-4}$ $=\dfrac{-4}{0}=$ undies	$f(-1)=\dfrac{(-1)^2+(-1)-6}{(-1)^2-4}$ $=\dfrac{1-7}{1-4}$ $=\dfrac{-6}{-3}=2$	x = -2 is not in the domain b/c when x = -2 there is not y-value. x = -1 is in the domain b/c when x = -1, y = 2
$g(x)=\sqrt{5-x}$	$g(-4)=\sqrt{5-(-4)}$ $=\sqrt{9}$ $=3$	$g(9)=\sqrt{5-(9)}$ $=\sqrt{-4}$ = undies	x = -4 is in the domain b/c when x = -4, y = 3. x = 9 is not in domain b/c when x = 9 there is no y-value
$h(x)=\sqrt[3]{x+3}$	$h(-11)=\sqrt[3]{(-11)+3}$ $=\sqrt[3]{-8}$ $=-2$	$h(5)=\sqrt[3]{(5)+3}$ $=\sqrt[3]{8}$ $=2$	x = -11 and x = 5 are in the domain b/c when x = -11, y = -2 and when x = 5, y = 2

Rational Functions

$f(x) = \dfrac{g(x)}{h(x)}$

Based on your observations of $f(x)$ in the table on page 46, what conclusion can you make about how to determine the domain of a rational function.

The domain of a rational Function is $(-\infty,\infty)$ EXCEPT for any x-values that make denominator $= 0$.

1. $f(x) = \dfrac{(2x-3)(x+2)}{(x-3)(x-1)}$

Denom $\neq 0$

$x-3 \neq 0$ | $x-1 \neq 0$

$x \neq 3$ | $x \neq 1$

Domain

$(-\infty,1)\cup(1,3)\cup(3,\infty)$

2. $g(x) = \dfrac{3x+2}{x^2-9} = \dfrac{3x+2}{(x-3)(x+3)}$

Denom $\neq 0$

$x-3 \neq 0$ | $x+3 \neq 0$

$x \neq 3$ | $x \neq -3$

Domain

$(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

3. $h(x) = \dfrac{x^2-x-2}{2x^2-5x-3} = \dfrac{(x-2)(x+1)}{(2x+1)(x-3)}$

Denom $\neq 0$

$2x+1 \neq 0$ | $x-3 \neq 0$

$2x \neq -1$ | $x \neq 3$

$x \neq -1/2$

Domain

$(-\infty,-\tfrac{1}{2})\cup(-\tfrac{1}{2},3)\cup(3,\infty)$

Square Root Functions

$f(x) = \sqrt{g(x)}$

$\sqrt{\text{RADICAND}}$

Based on your observations of $g(x)$ in the table on page 46, what conclusion can you make about how to determine the domain of a square root function.

The domain of a square root function will be the interval of x-values that makes RADICAND ≥ 0.

1. $f(x) = \sqrt{x+3}$

RADICAND ≥ 0

$x+3 \geq 0$

$x \geq -3$

Domain

$[-3,\infty)$

2. $g(x) = \sqrt{4-2x}$

RADICAND ≥ 0

$4-2x \geq 0$

$-2x \geq -4$

$x \leq 2$

Domain

$(-\infty,2]$

3. $h(x) = \sqrt{3x+1}$

RADICAND ≥ 0

$3x+1 \geq 0$

$3x \geq -1$

$x \geq -1/3$

Domain

$[-\tfrac{1}{3},\infty)$

Cubed Root Functions $f(x) = \sqrt[3]{g(x)}$	Based on your observations of $h(x)$ in the table on page 46, what conclusion can you make about how to determine the domain of a cubed root function. The domain of a cubed root function is $(-\infty,\infty)$ unless the radicand is rational (Denom $\neq 0$)

1. $f(x)=\sqrt[3]{x-3}$ Domain $(-\infty,\infty)$	2. $g(x)=\sqrt[3]{4-2x}$ Domain $(-\infty,\infty)$	3. $h(x)=\sqrt[3]{\frac{x-3}{2x+1}}$ Denom $\neq 0$ $2x+1\neq 0$ $2x\neq -1$ $x\neq -1/2$ Domain $(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},\infty)$