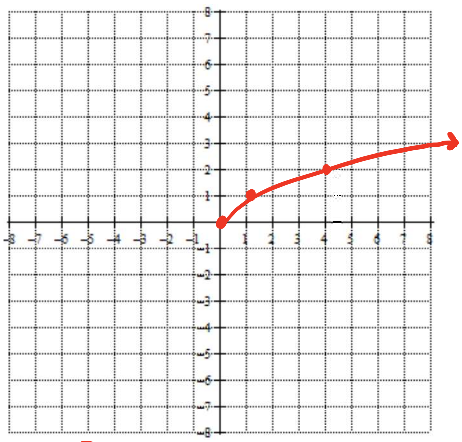
Notes 1.1 Shifting, Reflecting, and Stretching Graphs of Functions
Graphical and Analytical Approaches

Many functions can be graphed by simply knowing the general shape of basic graphs of functions and understanding what effect adding, subtracting and negating certain parts of the function have on the graph. Graph each of the functions below on the graphing calculator and then state how the change in the equations for Graphs B through I affected Graph A.

A. Basic Square Root Function

x	$\sqrt{x}$
0	0
1	1
4	2
9	3

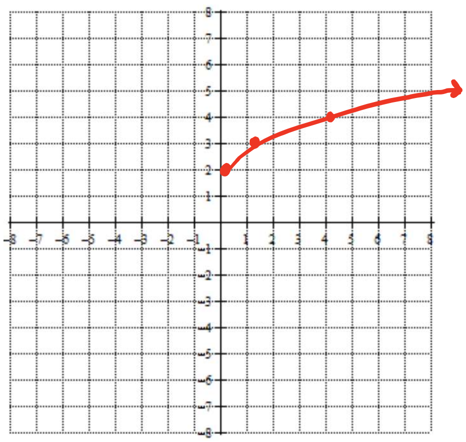
$f(x) = \sqrt{x}$



Parent Graph

B. Add 2 on the outside

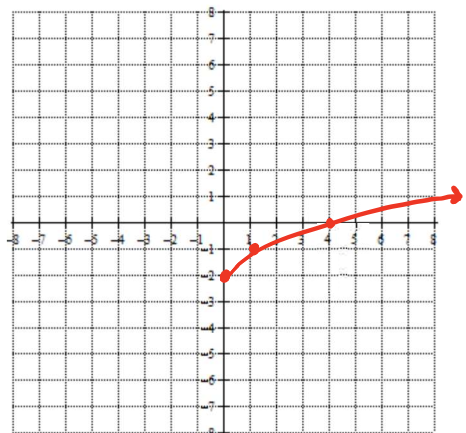
$f(x) = \sqrt{x} + 2$



Translate up 2

C. Subtract 2 on the outside

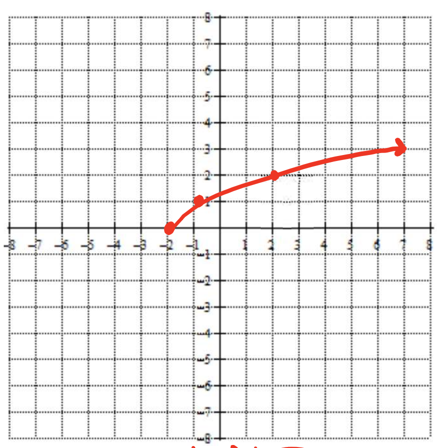
$f(x) = \sqrt{x} - 2$



Translate Down 2

D. Add 2 on the inside

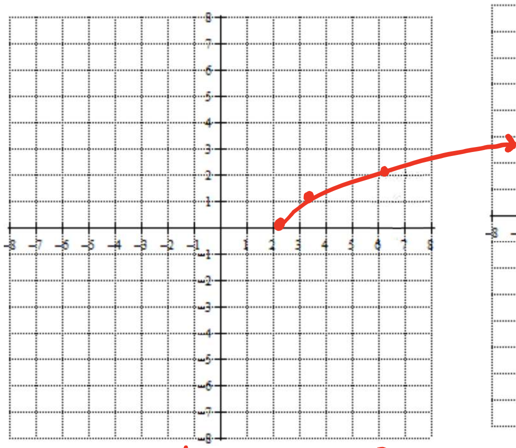
$f(x) = \sqrt{x+2}$



Translate left 2

E. Subtract 2 on the inside

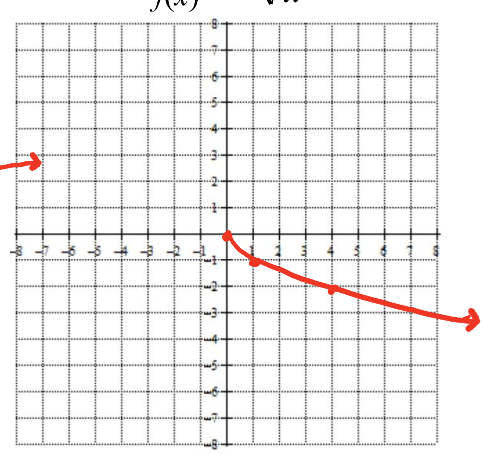
$f(x) = \sqrt{x-2}$



Translate right 2

F. Multiply the outside by -1

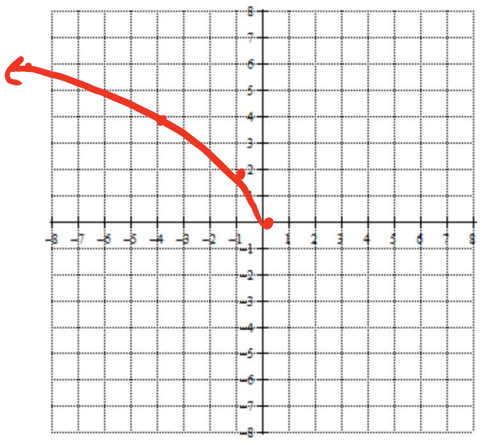
$f(x) = -\sqrt{x}$



Vertical Reflection

G. Multiply the outside by 2 AND Multiply the x by -1

$f(x) = 2\sqrt{-x}$

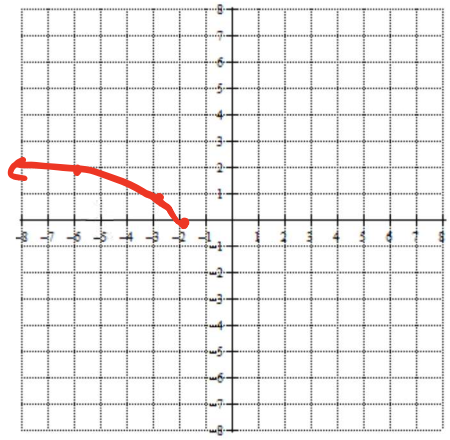


Horizontal reflection

vertical dilation by 2

H. Multiply the x by -1 AND Subtract 2 on the inside

$f(x) = \sqrt{-x-2} = \sqrt{-(x+2)}$

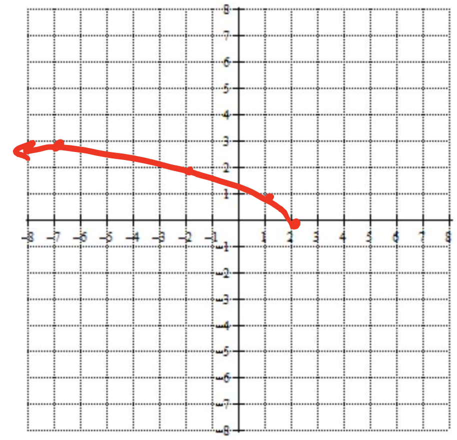


Horizontal Reflection

Translate left 2.

I. Multiply the x by -1 AND Add 2 on the inside.

$f(x) = \sqrt{-x+2} = \sqrt{-(x-2)}$



Horizontal Reflection

Translate right 2.

Based on your observations, predict how each of the graphs of the functions below would be different from the graph of $f(x) = \sqrt{x}$. State the changes that would be made to the basic graph. Then, state the domain and range of the function. The coefficient of x must be 1

$g(x) = 2\sqrt{-x+1} - 3$

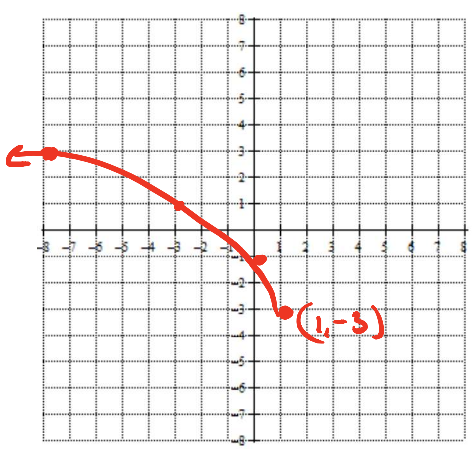
$g(x) = 2\sqrt{-(x-1)} - 3$

1) Reflect Horizontal

2) Dilate vertical by 2

3) Translate Horizontal right 1

vertical Down 3



Domain: $(-\infty, 1]$

Range: $[-3, \infty)$

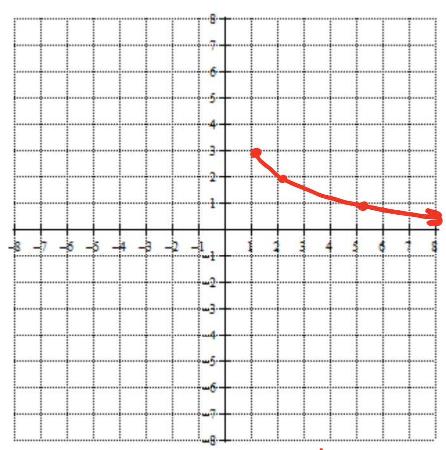
$h(x) = -\sqrt{x-1} + 3$

1) Reflect

2) Dilate

3) Translate Horizontal right 1

vertical up 3



Domain: $[1, \infty)$

Range: $(-\infty, 3]$

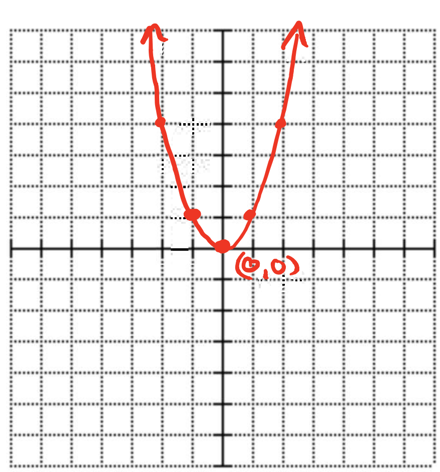
Complete the table below based on the observations that you have seen in the previous examples.

Equation with Transformations	Describe the shift(s) and/or reflections that the graph of $f(x)$ undergoes	Describe what would be done to the x and/or y coordinates to the graph of $f(x)$
$y = f(x) + c$	vertical shift up "c"	$(x, y+c)$
$y = f(x) - c$	vertical shift down "c"	$(x, y-c)$
$y = f(x + c)$	horizontal Shift left "c"	$(x-c, y)$
$y = f(x - c)$	horizontal Shift right "c"	$(x+c, y)$
$y = -f(x)$	vertical reflection	$(x, -y)$
$y = f(-x)$	horizontal reflection	$(-x, y)$
$y = \lvert f(x) \rvert$	Vertical reflection any portion of graph below x-axis	$(x, \lvert y \rvert)$
$y = a \cdot f(x)$	vertical dilation by factor of "a"	$(x, a \cdot y)$
$y = f(a \cdot x)$	horizontal dilation by factor of "$\frac{1}{a}$"	$(\frac{1}{a} \cdot x, y)$

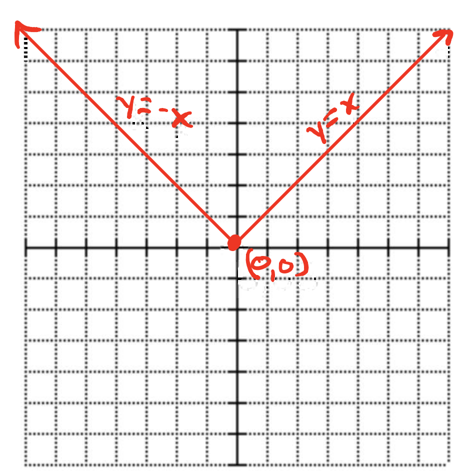
In Algebra II, you learned how to graph several other functions. Graph the parent functions below.

I. Basic Quadratic Function: $f(x) = x^2$

x	x^2
0	0
1	1
2	4
3	9

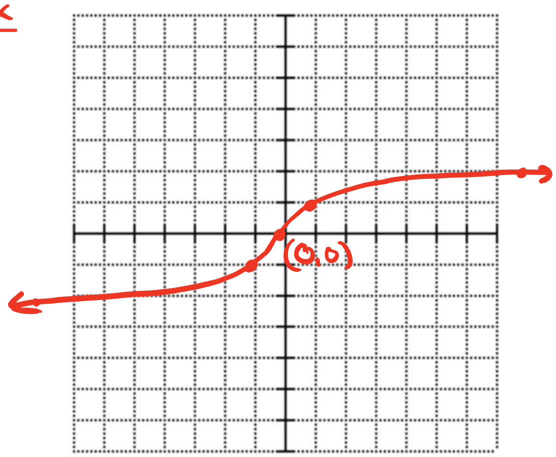


II. Basic Absolute Value Function: $f(x) = \lvert x \rvert$

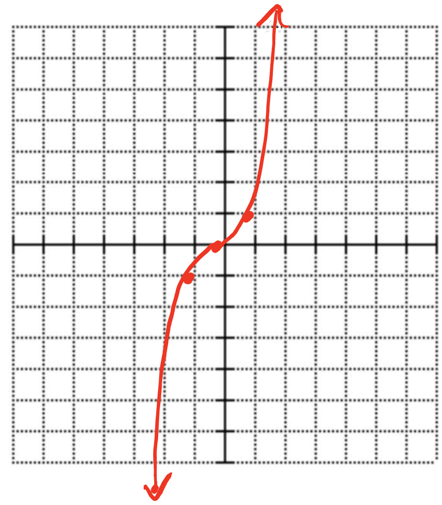


III. Basic Cubed Root Function: $f(x) = \sqrt[3]{x}$

x	$\sqrt[3]{x}$
0	0
1	1
8	2



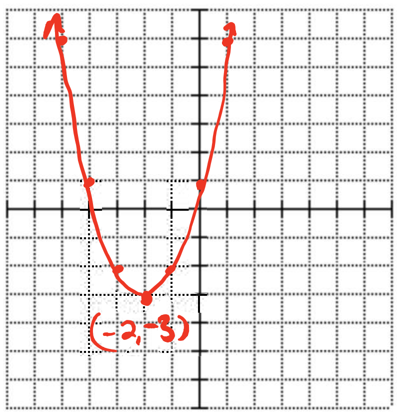
IV. Basic Cubic Function: $f(x) = x^3$



Describe how the graphs of each of the following functions will be different from the parent function. Then, graph the given functions

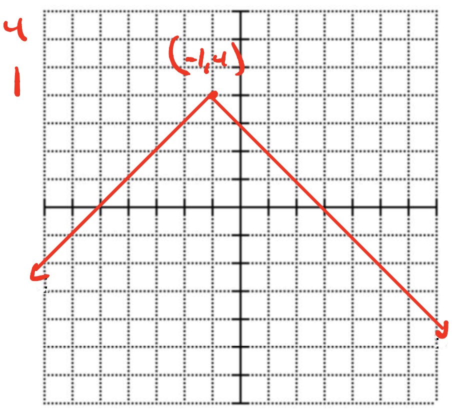
$g(x) = (x+2)^2 - 3$

Translate
- Horizontal left 2
- vertical Down 3



$h(x) = -|x+1| + 4$

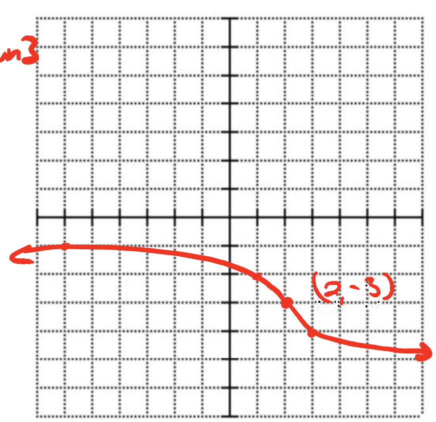
Reflect vertically
Translate up 4
left 1



$f(x) = \sqrt[3]{-x+2} - 3$

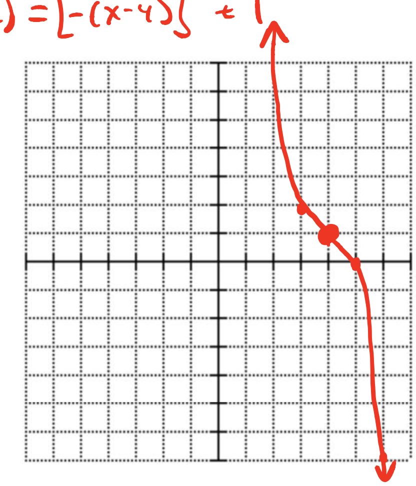
$= \sqrt[3]{-(x-2)} - 3$

- Reflect Horizontally
- Translate right 2, down 3



$p(x) = (-x+4)^3 + 1$

$p(x) = [-(x-4)]^3 + 1$



The table below shows ordered pairs on the graph of a function, $f(x)$, that consists of line segments connecting the points in the table. Use the table to create a table of values for each function below that is a transformation of the graph of $f(x)$.

x	–3	–1	1	3	5
$f(x)$	5	1	–4	1	2

1. $g(x) = -f(x) + 2$

State the transformations $f(x)$ undergoes to obtain the graph of $g(x)$, stating what changes are made to which coordinates of $f(x)$ to obtain the coordinates of point for $g(x)$.

① vertical reflection

② vertical shift up 2

$(x, -y+2)$

Coordinates on $f(x)$	x coordinate on $g(x)$ $x' = x$	y coordinate on $g(x)$ $y' = -y+2$	Ordered Pairs on $g(x)$
(–3, 5)	–3	$-5+2=-3$	(–3, –3)
(–1, 1)	–1	$-1+2=1$	(–1, 1)
(1, –4)	1	$4+2=6$	(1, 6)
(3, 1)	3	$-1+2=1$	(3, 1)
(5, 2)	5	$-2+2=0$	(5, 0)

2. $h(x) = 3f(x+2) - 3$

State the transformations that $f(x)$ undergoes to obtain the graph of $h(x)$, stating what changes are made to which coordinates of $f(x)$ to obtain the coordinates of point for $h(x)$.

horizontal
- shift left 2.

vertical
- dilation by factor of 3
- shift down 3

$(x-2, 3y-3)$

Coordinates on $f(x)$	x coordinate on $h(x)$	y coordinate on $h(x)$	Ordered Pairs on $h(x)$
(–3, 5)	$-3-2=-5$	$3(5)-3=12$	(–5, 12)
(–1, 1)	$-1-2=-3$	$3(1)-3=0$	(–3, 0)
(1, –4)	$1-2=-1$	$3(-4)-3=-15$	(–1, –15)
(3, 1)	$3-2=1$	$3(1)-3=0$	(1, 0)
(5, 2)	$5-2=3$	$3(2)-3=3$	(3, 3)

3. $q(x) = f(-x+3) - 2$

State the transformations that $f(x)$ undergoes to obtain the graph of $q(x)$, stating what changes are made to which coordinates of $f(x)$ to obtain the coordinates of point for $q(x)$.

$q(x) = f[-(x-3)] - 2$

Horizontal
- reflection
- shift right 3

vertical
- shift down 2

$(-x+3, y-2)$

Coordinates on $f(x)$	x coordinate on $q(x)$	y coordinate on $q(x)$	Ordered Pairs on $q(x)$
(–3, 5)	$3+3=6$	$5-2=3$	(6, 3)
(–1, 1)	$1+3=4$	$1-2=-1$	(4, –1)
(1, –4)	$-1+3=2$	$-4-2=-6$	(2, –6)
(3, 1)	$-3+3=0$	$1-2=-1$	(0, –1)
(5, 2)	$-5+3=-2$	$2-2=0$	(–2, 0)

Graphs of Piece-wise Defined Functions:

Graph the following piecewise defined functions on the provided grids, state the domain and range.

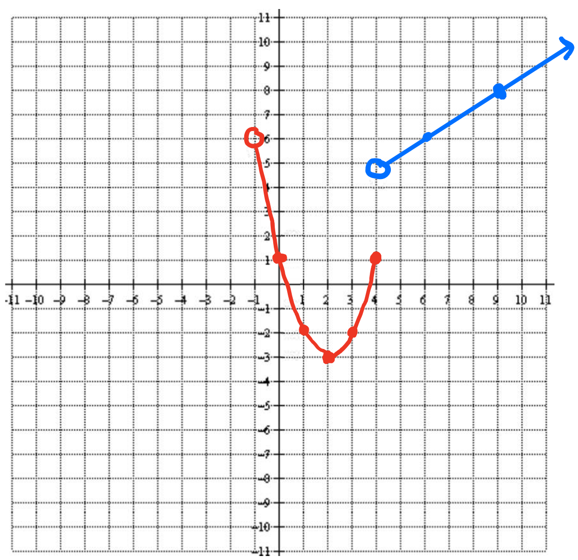
1. $g(x)=\begin{cases}(x-2)^2-3, & -1<x\le 4\\ \frac{2}{3}x+2, & x>4\end{cases}$ vertex $(2,-3)$

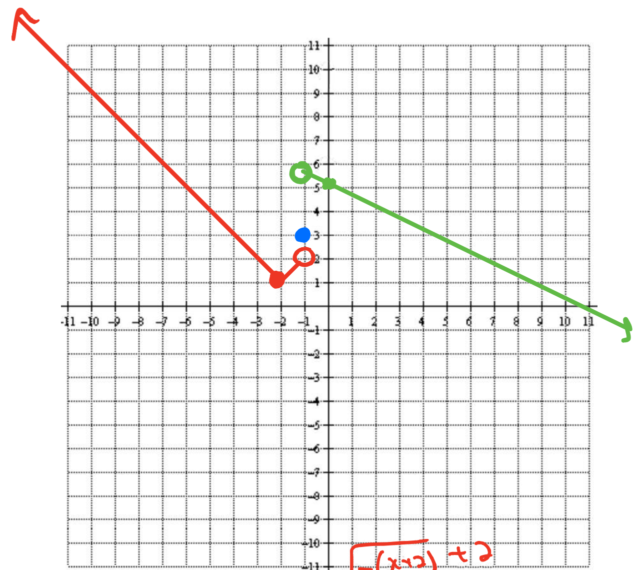
$g(9)=\frac{2}{3}(9)+2$
$=6+2$

D: $(-1,\infty)$ R: $(-3,\infty)$

2. $h(x)=\begin{cases}|x+2|+1, & x<-1\\ 3, & x=-1\\ -\frac{1}{2}x+5, & x>-1\end{cases}$ vertex $(-2,1)$ Point $(-1,3)$ y-int 5

D: $(-\infty,\infty)$ R: $(-\infty,\infty)$





$\sqrt{-(x+2)}+2$

3. $g(x)=\begin{cases}\sqrt{-x}+3, & -4\le x<0\\ 2, & x=0\\ \sqrt{x}+3, & x>0\end{cases}$ vertex $(0,3)$ Point $(0,2)$ vertex$(0,3)$

D: $[-4,\infty)$
R: $y=2$ and $(3,\infty)$

4. $p(x)=\begin{cases}\sqrt{-x-2}+2, & x<-2\\ -2x-2, & -2<x<1\\ -\sqrt{x-1}-4, & x>1\end{cases}$ vertex $(-2,2)$ y-int -2 vertex$(1,-4)$

D: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$
R: $(-\infty,-4)\cup(-4,2)\cup(2,\infty)$