Review Quiz 8

FRQ 1: Calculator Permitted

Year	Population
0	75
1	153
2	312
3	725
4	1523

The population of a colony of rabbits is growing exponentially. The table above shows the population of rabbits at the beginning of each year, t, where $t = 0$ represents the beginning of calendar year of 2016.

a. Using the regression capabilities of a graphing calculator, find a function, $f(t) = a \cdot b^t$, that represents the population of the colony at the end of a given year, t. Based on the equation, why does the formula represent an exponential growth?

L_1	L_2
0	75
1	153
2	312
3	725
4	15...
---...	---...

Xmin = 0
Xmax = 10
Xscl = 1
Ymin = 0
Ymax = 3000
Yscl = 50
Xres = 1

ExpReg
X List : L_1
Y List : L_2
[Freq List] : □
[Store RegEQ] : Y_1
$y = a \cdot b^x$
$a = 72.6447558506$
$b = 2.1334987147$
$r^2 = 0.9990525602$
$r = 0.9995261678$

Plot 1 Plot 2 Plot 3
$\backslash Y_1 = 8506 \cdot 2.1334987147^X$

+1 $f(t) = 72.645 \cdot (2.133)^t$

+1 $a > 0$, $c > 0$ $\}$ ∴ no reflections

+1 ∴ f(t) has exponential growth

b. Find the value of $f(7)$ and, using a calendar year, tell what this value represents in terms of the population of the colony.

$f(7) = 14,616.755$ +1

At the beginning of year 2023, +1

there will be 14,617 rabbits +1

(or 14,616)

t	Beginning of Year
0	2016
1	17
2	18
3	19
4	20
5	21
6	22
7	2023

X	Y_1
7	1461...

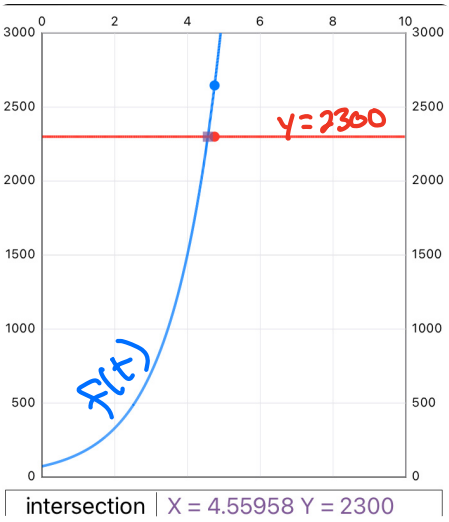
$Y_1 = 14616.7549952778$

c. When the population reaches 2300, the colony will need to be split because excessive inbreeding will cause genetic mutations that could harm the health of the offspring. During what calendar year will the colony need to be split? Show and explain the analysis that leads to your answer.

$f(t) = 72.645 \cdot (2.133)^t$

+1 $2300 = 72.645 \cdot (2.133)^t$

$t \approx 4.560$



$t = 4$ represents beginning of 2020

$t = 5$ represents beginning of 2021

+1 The population will be split during the year 2020.

MULTIPLE CHOICE – Calculator Permitted

1. Which of the following equations represent(s) an exponential function that increases with bound?

I. $f(x) = -(5)^x$ II. $g(x) = -3 - (0.25)^{x+2}$ III. $h(x) = 2(1.5)^{-x-1}$

$g(x) = -3 - (4)^{-(x+2)}$

A. I only B. II only C. III only

D. II and III only E. I, II and III

2. Identify the range of the exponential function $f(x) = \left(\frac{5}{4}\right)^{-(x+5)} + 3$.

HA $y = 3$

A. $(-\infty, 5)$ B. $(-\infty, 3)$ C. $(3, \infty)$

D. $(5, \infty)$ E. $(-5, \infty)$

$-(x+5)$

3. The point (0, 1) is on the graph of $f(x) = e^x$. What is the corresponding point on the graph of $g(x)$ if $g(x) = e^{-x-5} + 2$?

A. (–5, 3) B. (–5, 2) C. (2, 5)

D. (5, 3) E. (2, -5)

$(x, y) \rightarrow (-x-5, y+2)$

$(0, 1) \rightarrow (-5, 3)$

Check $g(-5) = e^{-(-5)-5} + 2$

$= e^0 + 2$

$= 1 + 2$

$g(-5) = 3$ ✓

4. Solve the equation $3^x \cdot \left(\frac{1}{9}\right)^{-2x+3} = 27^{x+9}$

A. $x = -2$

B. $x = \frac{33}{2}$

C. $x = -\frac{33}{2}$

D. $x = -\frac{2}{33}$

E. None of these

$3^x \cdot 9^{2x-3} = 3^{3x+27}$

$3^x \quad 3^{4x-6} = 3^{3x+27}$

$3^{5x-6} = 3^{3x+27}$

$5x - 6 = 3x + 27$

$2x = 33$

$x = \frac{33}{2}$

5. What is the equation of the horizontal asymptote of the exponential function $f(x) = -3^{x+5} - 7$?

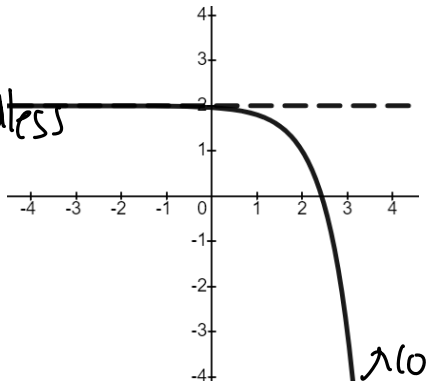
A. $y = 5$ B. $y = -5$ C. $y = 7$ D. $y = -7$ E. $y = 0$

6. Consider the exponential function, $f(x) = a \cdot b^{x-2} + k$ graphed to the right (assuming b > 1). Which of the following statements is/are true?

I. The value of k is 2. True (HA @ $y=2$)

II. As $x \to \infty$, $f(x)$ decreases with bound. False, boundless

III. The value of a is such that $a > 0$. False, f is below

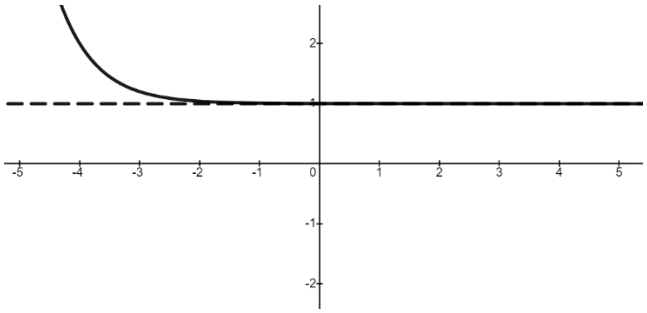


↗ to boundary

A. I only B. II only C. III only

D. I and II only E. I, II and III

7. The graph of an exponential function, $f(x) = a \cdot b^{c(x+4)} + k$ is pictured to the right (assuming b > 1). Which of the following statements is/are true?



f is above goes toward H.A @ y=1

I. The value of $a < 0$. False

II. The value of $c < 0$. True

III. The value of $k = 2$. False

A. I and II only B. II only C. I, II, and III

D. II and III only E. I and III only

8. Simply the following expression: $5^{12-2x} \cdot 25^{x-6}$

A. 1 B. 5^{-x+6} C. 5^{4x-9}

D. 25^{-x+6} E. None of these

$5^{12-2x}\ 5^{2x-12} = 5^0 = 1$

9. Which of the following conclusions can be made (assuming b > 1) in the function, $g(x) = a \cdot b^{c(x-h)} + k$, pictured to the right?

A. $c < 0$ True

B. $a \cdot c > 0$ True

C. $a > 0$ False

D. Both A and B

E. Both B and C False

$c < 0$

$a < 0$