Homework 8.4

Complete the table for each of the exponential functions below. Be sure to give justification when asked to do so.

Function	What are the domain, range, and horizontal asymptote of the exponential function?	Is the graph of the function above or below the horizontal asymptote? Why?	What are the domain, range, and the equation of the vertical asymptote of the inverse function?	Is the graph of the inverse function to the left or right of the vertical asymptote? Why?
1. $F(x) = 2^{x-3}$	D: $(-\infty, \infty)$ R: $(0, \infty)$ HA: $y = 0$	$a > 0$ ∴ the graph of $F(x)$ is above HA.	D: $(0, \infty)$ R: $(-\infty, \infty)$ VA: $x = 0$	D: $(0, \infty)$ ∴ the graph of $F^{-1}(x)$ is to the right of VA
2. $G(x) = -\left(\frac{1}{2}\right)^{x+3} - 1$ $G(x) = -(2)^{-(x+3)} - 1$	D: $(-\infty, \infty)$ R: $(-\infty, -1)$ HA: $y = -1$	$a < 0$ ∴ the graph of $G(x)$ is below HA.	D: $(-\infty, -1)$ R: $(-\infty, \infty)$ VA: $x = -1$	D: $(-\infty, -1)$ ∴ the graph of $G^{-1}(x)$ is to the left of VA
3. $H(x) = (1.25)^{-x+2} + 3$ $H(x) = (1.25)^{-(x-2)} + 3$	D: $(-\infty, \infty)$ R: $(3, \infty)$ HA: $y = 3$	$a > 0$ ∴ the graph of $H(x)$ is above HA.	D: $(3, \infty)$ R: $(-\infty, \infty)$ VA: $x = 3$	D: $(3, \infty)$ ∴ the graph of $F^{-1}(x)$ is to the right of VA
4. $f(x) = \left(\frac{3}{2}\right)^{-x-1} + 2$ $f(x) = \left(\frac{3}{2}\right)^{-(x+1)} + 2$	D: $(-\infty, \infty)$ R: $(2, \infty)$ HA: $y = 2$	$a > 0$ ∴ the graph of $f(x)$ is above HA.	D: $(2, \infty)$ R: $(-\infty, \infty)$ VA: $x = 2$	D: $(2, \infty)$ ∴ the graph of $F^{-1}(x)$ is to the right of VA

For each of the functions below, find the equation of the inverse function. Show your work.

5. $F(x) = 2^{x-3}$

$x = 2^{y-3}$ (Exp Form)

$\log_2 x = y - 3$ (Log Form)

$\log_2(x) + 3 = y$

$F^{-1}(x) = \log_2(x) + 3$

6. $G(x) = 2e^{x-2} + 4$

$x = 2e^{y-2} + 4$

$x - 4 = 2e^{y-2}$

$\frac{1}{2}(x-4) = e^{y-2}$ (Exp Form)

$\ln\left[\frac{1}{2}(x-4)\right] = y - 2$ (Log Form)

$\ln\left[\frac{1}{2}(x-4)\right] + 2 = y$

$G^{-1}(x) = \ln\left[\frac{1}{2}(x-4)\right] + 2$

7. $H(x) = 3^{-x+3} - 1$

$x = 3^{-y+3} - 1$

$x + 1 = 3^{-y+3}$ (Exp Form)

$\log_3(x+1) = -y + 3$ (Log Form)

$\log_3(x+1) - 3 = -y$

$-\log_3(x+1) + 3 = y$

$H^{-1}(x) = -\log_3(x+1) + 3$

For each of the logarithmic functions below, state the equation of the vertical asymptote. Also, state the domain and range. Show your work.

8. $G(x) = \log_2(3 - 2x) + 2$

Domain: Argument > 0

$3 - 2x > 0$

$-2x > -3$

$x < 3/2$

VA at $x = 3/2$

D: $(-\infty, 3/2)$

R: $(-\infty, \infty)$

9. $F(x) = -2 + \ln(-x - 3)$

Domain: Argument > 0

$-x - 3 > 0$

$-x > 3$

$x < -3$

VA at $x = -3$

D: $(-\infty, -3)$

R: $(-\infty, \infty)$

10. $H(x) = -\log_3\left(\frac{1}{2}x - 3\right)$

Domain: Argument > 0

$\frac{1}{2}x - 3 > 0$

$\frac{1}{2}x > 3$

$x > 6$

VA at $x = 6$

D: $(6, \infty)$

R: $(-\infty, \infty)$

Consider the function $g(x) = -(2)^{x-3} - 2$ to answer questions 11 – 15.

11. Explain how the graph of $g(x) = -(2)^{x-3} - 2$ is different from the graph of $f(x) = (2)^x$.

Compared to f(x) . . .

① g(x) is reflected over the HA, y = -2

② g(x) is translated right 3 and down 2

12. Based on the transformations you described in exercise 11, complete the following table of values.

x	$f(x)$	Coordinate Points of $g(x)$ $(x+3, -y-2)$	Coordinate Points of $g^{-1}(x)$
–2	$\frac{1}{4}$	$(1, -2.25)$	$(-2.25, 1)$
–1	$\frac{1}{2}$	$(2, -2.5)$	$(-2.5, 2)$
0	1	$(3, -3)$	$(-3, 3)$
1	2	$(4, -4)$	$(-4, 4)$
2	4	$(5, -6)$	$(-6, 5)$

13. Find the equation of $g^{-1}(x)$.

$x = -(2)^{y-3} - 2$

$x + 2 = -(2)^{y-3}$

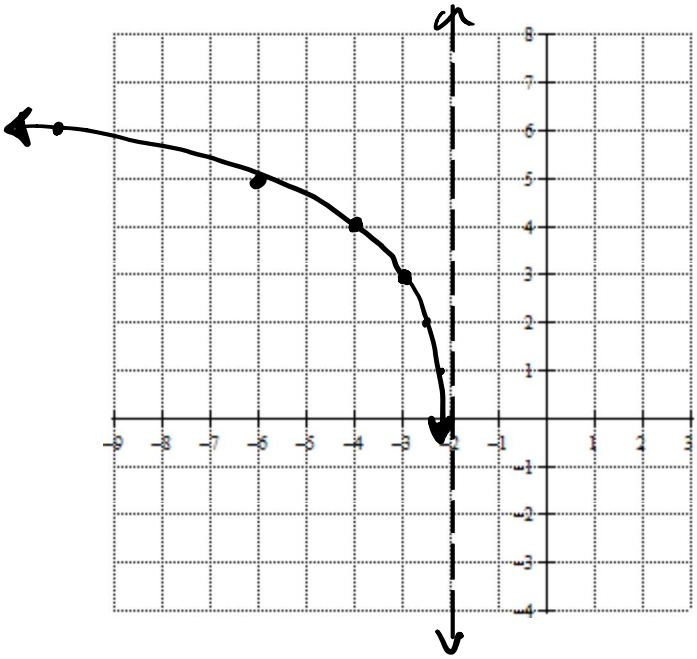
$-(x+2) = 2^{y-3}$ (Exp Form)

$\log_2[-(x+2)] = y - 3$ (Log Form)

$\log_2[-(x+2)] + 3 = y$

$g^{-1}(x) = \log_2[-(x+2)] + 3$

14. Sketch the graph of $g^{-1}(x)$.



15. Domain of $g^{-1}(x)$: $(-\infty, -2)$

Range of $g^{-1}(x)$: $(-\infty, \infty)$

Below is a table of values for the exponential function $f(x) = e^{x-2} - 3$. Use the equation of $f(x)$ and the table of values to answer the questions that follow.

x	–5	–2	0	2	4	6
$f(x)$	–2.999	–2.982	–2.865	–2	4.389	51.598

16. Fill in the table below identifying the domain, range, and asymptotes of the graphs of $f(x)$ and $f^{-1}(x)$.

	$f(x)$		$f^{-1}(x)$
Domain	$(-\infty, \infty)$	Domain	$(-3, \infty)$
Range	$(-3, \infty)$	Range	$(-\infty, \infty)$
Horizontal Asymptote	$y = -3$	Vertical Asymptote	$x = -3$

17. Is the graph of $f^{-1}(x)$ to the right or left of the vertical asymptote that you identified above? Give a reason for your answer.

The domain of $f^{-1}(x)$ is $(-3, \infty)$

$\therefore$ the graph of $f^{-1}(x)$ is to the right of VA

18. Find the equation of $f^{-1}(x)$. Then, use the equation to find the equation of the vertical asymptote.

$x = e^{y-2} - 3$

$x + 3 = e^{y-2}$

$\ln(x+3) = y - 2$

$\ln(x+3) + 2 = y$

$f^{-1}(x) = \ln(x+3) + 2$

19. Sketch a graph of $f^{-1}(x)$ on the grid to the right.