

Homework 8.3 Part 1

Simplify the following expressions using the inverse properties of logarithms and exponentials.

1. $\ln e^6 = 6$	2. $5^{\log_5 3} = 3$	3. $e^{\ln x + 2} = e^{\ln x} \cdot e^2 = x e^2$	4. $3^{2 \log_3 5} = 3^{\log_3 5^2} = 3^{\log_3 25} = 25$
5. $\log_5(5\sqrt{5}) = \log_5(5 \cdot 5^{1/2}) = \log_5(5^{3/2}) = 3/2$	6. $e^{\ln(x+2)+3} = e^{\ln(x+2)} \cdot e^3 = (x+2)e^3 = x e^3 + 2e^3$	7. $\ln e^{x+2} = x+2$	8. $3^{\log_3(x+3)+3} = 3^{\log_3(x+3)} \cdot 3^3 = (x+3) 27 = 27x + 81$

Solve each of the following exponential equations. Give your answer in both exact form and rounded to three decimal places, if necessary.

9. $2^{x+3} - 5 = 3$
 $2^{x+3} = 8$
 $2^{x+3} = 2^3$
 $x+3 = 3$
 $x = 0$

10. $3^{2x} = 5^{x+3}$
 $\log 3^{2x} = \log 5^{x+3}$
 $2x \log 3 = (x+3) \log 5$
 $2x \log 3 = x \log 5 + 3 \log 5$
 $2x \log 3 - x \log 5 = 3 \log 5$
 $x(2 \log 3 - \log 5) = 3 \log 5$
 $x = \frac{3 \log 5}{2 \log 3 - \log 5}$
 $x \approx 8.214$

11. $3e^{2x} - 10e^x - 8 = 0$
 $3e^{2x} - 12e^x + 2e^x - 8 = 0$
 $3e^x(e^x - 4) + 2(e^x - 4) = 0$
 $(e^x - 4)(3e^x + 2) = 0$
 $e^x - 4 = 0$
 $e^x = 4$
 $\ln e^x = \ln 4$
 $x = \ln 4$
 $x \approx 1.386$
 $3e^x + 2 = 0$
 $3e^x = -2$
 $e^x = -2/3$
 $\ln e^x = \ln(-2/3)$
 $x = \ln(-2/3) \Rightarrow \text{undefined}$
M = -24e^{2x}
A = -10e^x
N = -12e^x, 2e^x

12. $7 + 3e^{2-3x} = 10$
 $3e^{2-3x} = 3$
 $e^{2-3x} = 1$
 $\ln e^{2-3x} = \ln 1$
 $2-3x = 0$
 $2 = 3x$
 $2/3 = x$
 $0.667 \approx x$
 *$\ln 1 = \log_e 1$
 which means $e^x = 1$
 $x = 0$*

13. Solve $3 - 2e^{2x+3} = 11$ for x . Explain what happens that causes this equation to have no solution.

$$\begin{aligned}
 -2e^{2x+3} &= 8 \\
 e^{2x+3} &= -4 \\
 \ln e^{2x+3} &= \ln(-4) \\
 2x+3 &= \ln(-4)
 \end{aligned}$$

No solution b/c the argument of a logarithm must be > 0 .

Solve each of the following logarithmic equations. Give your answer in both exact form and rounded to three decimal places, if necessary.

14. $\log_3(x+2) + \log_3 4 = 2$

$$\begin{aligned}
 \log_3(4x+8) &= 2 \quad (\text{log Form}) \\
 3^2 &= 4x+8 \quad (\text{Exp Form}) \\
 9 &= 4x+8 \\
 1 &= 4x \\
 \frac{1}{4} &= x
 \end{aligned}$$

15. $\ln(x-3) - \ln(x+1) = \ln\left(\frac{1}{2}\right)$

$$\begin{aligned}
 \ln \frac{x-3}{x+1} &= \ln\left(\frac{1}{2}\right) \\
 \frac{x-3}{x+1} &= \frac{1}{2} \\
 2(x-3) &= x+1 \\
 2x-6 &= x+1 \\
 x-6 &= 1 \\
 x &= 7
 \end{aligned}$$

16. $\ln(2x+3) + 2 = 5$

$$\begin{aligned}
 \ln(2x+3) &= 3 \quad (\text{log Form}) \\
 e^3 &= 2x+3 \quad (\text{Exp Form}) \\
 e^3 - 3 &= 2x \\
 \frac{1}{2}(e^3 - 3) &= x \\
 8.543 &\approx x
 \end{aligned}$$

17. $\log_2(3-2x) = -2$

$$\begin{aligned}
 2^{-2} &= 3-2x \quad (\text{Exp Form}) \\
 \frac{1}{4} &= 3-2x \\
 1 &= 12-8x \\
 -11 &= -8x \\
 \frac{11}{8} &= x \\
 1.375 &\approx x
 \end{aligned}$$

18. Solve $\ln(x+2) - \ln(x-1) = \ln\left(\frac{2}{3}\right)$ for x . Explain why there is no solution to this equation.

$$\begin{aligned}
 \ln \frac{x+2}{x-1} &= \ln\left(\frac{2}{3}\right) \\
 \frac{x+2}{x-1} &= \frac{2}{3} \\
 3(x+2) &= 2(x-1) \\
 3x+6 &= 2x-2 \\
 x+6 &= -2 \\
 x &= -8
 \end{aligned}$$

The argument of a logarithm must be > 0 . If $x = -8$, then both arguments are negative.

Thus there is no solution.