Homework 8.2

Rewrite each of the following expressions as a single logarithm of the same base.

1. $5\log x + 2\log x$

$= \log x^5 + \log x^2$

$= \log(x^5 \cdot x^2)$

$= \log x^7$

2. $\log_3 125 - \log_3 5$

$= \log_3 \frac{125}{5}$

$= \log_3(25)$

3. $2\ln x + 3\ln y - 5\ln z$

$= \ln x^2 + \ln y^3 - \ln z^5$

$= \ln\left(\frac{x^2 y^3}{z^5}\right)$

4. $\log_4 60 - \log_4 4 + \log_4 x$

$= \log_4 \frac{60 \cdot x}{4}$

$= \log_4(15x)$

or

$\log_4 60 - \log_4 4 + \log_4 x$

$= \log_4 60 - 1 + \log_4 x$

$= \log_4(60x) - 1$

5. $\frac{1}{2}\ln x + 2\ln x$

$= \ln x^{\frac{1}{2}} + \ln x^2$

$= \ln(x^{1/2} x^2)$

$= \ln(x^{1/2} x^{4/2})$

$= \ln(x^{5/2})$

6. $\ln(4x) + 2\ln(2x)$

$= \ln(4x) + \ln(2x)^2$

$= \ln(4x) + \ln(4x^2)$

$= \ln(4x \cdot 4x^2)$

$= \ln(16x^3)$

Expand each of the following expressions as the sum and/or difference of multiple logarithms.

7. $\log(xyz)$ $= \log x + \log y + \log z$

8. $\ln\left(\frac{2x}{y}\right)$ $= \ln 2 + \ln x - \ln y$

9. $\ln(6x^2y)$ $= \ln 6 + \ln x^2 + \ln y$

$= \ln 6 + 2\ln x + \ln y$

10. $\log_2\left(\frac{7}{xy^2}\right)$ $= \log_2 7 - \log_2 x - \log_2 y^2$

$= \log_2 7 - \log_2 x - 2\log_2 y$

11. $\log_5 \sqrt{x^3 y} = \log_5 \left(x^{3/2} y^{1/2}\right)$

$= \log_5 x^{3/2} + \log_5 y^{1/2}$

$= \frac{3}{2}\log_5 x + \frac{1}{2}\log_5 y$

12. $\ln\left(\frac{3x}{y^2}\right) = \ln(3x) - \ln(y^2)$

$= \ln 3 + \ln x - 2\ln y$

Solve each of the following equations for x by rewriting each side of the equation as a single logarithm of the same base. Then, set the arguments equal to one another. If the equation has no solution, then specifically explain why.

13. $\log_3(x+2) - \log_3 2 = \log_3(2x-5)$ $x>-2$ $x>5/2$

$\log_3 \frac{x+2}{2} = \log_3(2x-5)$

$\frac{x+2}{2} = 2x-5$

$x+2 = 4x-10$

$2 = 3x-10$

$12 = 3x$

$4 = x$

14. $\ln(2x+5) - \ln 5 = \ln(x-2)$ $x>-5/2$ $x>2$

$\ln \frac{2x+5}{5} = \ln(x-2)$

$\frac{2x+5}{5} = x-2$

$2x+5 = 5x-10$

$5 = 3x-10$

$15 = 3x$

$5 = x$

15. $2\ln 3 + \ln(x-4) = \ln(3x)$ $x>4$ $x>0$

$\ln 3^2 + \ln(x-4) = \ln(3x)$

$\ln 9 + \ln(x-4) = \ln(3x)$

$\ln(9x-36) = \ln(3x)$

$9x-36 = 3x$

$6x-36 = 0$

$6x = 36$

$x = 6$

16. $\log(x-3) + \log(x) = \log 12 - \log 3$ $x>3$ $x>0$

$\log(x^2-3x) = \log\left(\frac{12}{3}\right)$

$x^2-3x = 4$

$x^2-3x-4 = 0$

$(x-4)(x+1) = 0$

$x-4=0$ $\quad x=4$ $\qquad$ $x+1=0$ $\quad x=-1$, but $x \neq -1$ b/c ...

$\log(x-3) + \log x$

$= \log(-1-3) + \log(-1)$

$= \log(-4) + \log(-1)$

The argument must be >0

$\therefore x = 4$ only