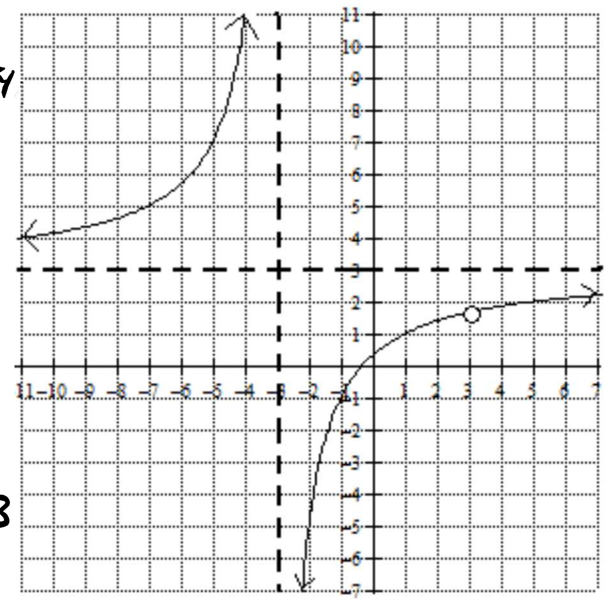
Homework 5.6

Answer questions 1 – 5 using the graph of the rational function, $f(x)$, pictured to the right. The graph of the function has a removable point discontinuity at $\left(3, \frac{5}{3}\right)$ and an x – intercept at $\left(-\frac{1}{3},\ 0\right)$.



1. At what value(s) of x does the graph of $f(x)$ have a non-removable, infinite discontinuity? Describe the behavior of $f(x)$ as x approaches this value of x from the left and the right.

f(x) has a non-removable, infinite discontinuity at $x = -3$.

$\lim_{x \to -3^-} f(x) = \infty$ $\lim_{x \to -3^+} f(x) = -\infty$

2. At what value(s) of x does the graph of $f(x)$ have a removable, point discontinuity? Describe the behavior of $f(x)$ as x approaches this value of x from the left and the right.

f(x) has removable, point discontinuity at $x=3$

$\lim_{x \to 3^-} f(x) = 5/3$ $\lim_{x \to 3^+} f(x) = 5/3$

3. What factors are guaranteed to be in the denominator of the equation? Give a reason for your answer.

f(x) is discontinuous at $x=-3$ and $x=3$

$\therefore$ $(x+3)$ and $(x-3)$ are guaranteed factors of f(x)'s denominator.

4. What factor is guaranteed to be in the numerator of the equation but not in the denominator? Give a reason for your answer.

f(x) has a zero at $x = -\frac{1}{3}$.

$\therefore$ $(3x+1)$ is guaranteed to be a factor in the numerator but not the denominator.

5. What are the domain and range of $f(x)$?

D: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

R: $(-\infty, 5/3) \cup (5/3, 3) \cup (3, \infty)$

The table below represents values on the graph of a rational function, $h(x) = \frac{2x^2+14x+24}{x^2-9}$.

x	−150	−11	−4	−3.002	−3	−2.997	2	2.998	3	3.002	4	150
$h(x)$	1.908	1	0	−0.333	Undefined	−0.335	−12	−6998	Undefined	7002	16	2.095

6. Based on the table, what factor is guaranteed to be in the numerator of the equation of the function but not in the denominator? Give a reason for your answer.

$h(x) = 0$ when $x = -4$. $\therefore h(x)$ has a zero at $x = -4$.

$\therefore (x+4)$ is a factor of the numerator but not the denominator.

7. At what x – value does the graph of $h(x)$ have a vertical asymptote? Give a reason for your answer based on the values in the table.

$$\left.\begin{aligned} \lim_{x \to 3^-} h(x) &= -\infty \\ \lim_{x \to 3^+} h(x) &= \infty \end{aligned}\right\} \therefore h(x) \text{ has a vertical asymptote at } x = 3$$

8. At what x – value does the graph of $h(x)$ have a point discontinuity? Give a reason for your answer based on the values in the table.

$$\left.\begin{aligned} \lim_{x \to -3^-} h(x) &= -\tfrac{1}{3} \\ \lim_{x \to -3^+} h(x) &= -\tfrac{1}{3} \\ h(-3) &\neq -\tfrac{1}{3} \end{aligned}\right\} \therefore h(x) \text{ has point discontinuity at } x = -3$$

9. Does the table indicate horizontally asymptotic behavior? If so, indicate the equation of the horizontal asymptote. Give a reason for your answer based on the values in the table.

$$\left.\begin{aligned} \lim_{x \to -\infty} h(x) &= 2 \\ \lim_{x \to \infty} h(x) &= 2 \end{aligned}\right\} \therefore h(x) \text{ has horizontal asymptote at } y = 2$$

10. Completely factor the equation of the function $h(x)$. What connections do you see between the factors of the equation and the conclusions that you made in questions 7 and 8?

$$h(x) = \frac{2x^2+14x+24}{x^2-9} = \frac{2(x^2+7x+12)}{(x-3)(x+3)} = \frac{2(x+4)(x+3)}{(x-3)(x+3)}$$

- $(x-3)$ is a non-canceling factor in the denominator so $x = 3$ is a vertical asymptote.
- $(x+3)$ is a canceling factor, so $x = -3$ is removable, point discontinuity.