Homework 5.5

Use the function $f(x) = \frac{x^2+x-6}{x^2-4}$ to answer questions 1 – 7.

1. What is the equation of the function written in completely factored form?

$f(x) = \frac{(x+3)(x-2)}{(x+2)(x-2)} \Longrightarrow f(x) = \frac{x+3}{x+2}$, hole at $x=2$

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2. If any exist, identify the vertical asymptotes? Explain how you know that they are vertical asymptotes.

f(x) has a non canceling factor of (x+2) in the denominator.
∴ f(x) has a VA at $x=-2$

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3. Does the function have any holes in the graph? Explain why or why not. What are the coordinates where the hole(s) exist(s)?

f(x)'s numerator and denominator share a factor of (x−2)
∴ f(x) has a hole at $x=2$
$f(x) = \frac{x+3}{x+2} \Longrightarrow f(2) = \frac{2+3}{2+2} = 5/4 \Longrightarrow$ Hole $(2, 5/4)$

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4. If any exist, identify the horizontal asymptotes. Explain how you know that they are horizontal asymptotes.

f(x)'s numerator and denominator have the same degree.
∴ f(x) has a horizontal asymptote at the ratio of lead coefficients, $y=\frac{1}{1}$
HA at $y=1$

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5. What is/are the zero(es) of the function? Show your work.

$x+3=0$
$x=-3$ is a zero of f(x)

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6. What are the domain and range of the function? Give your answer in interval notation.

Domain $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
Range $(-\infty,1)\cup(1,5/4)\cup(5/4,\infty)$

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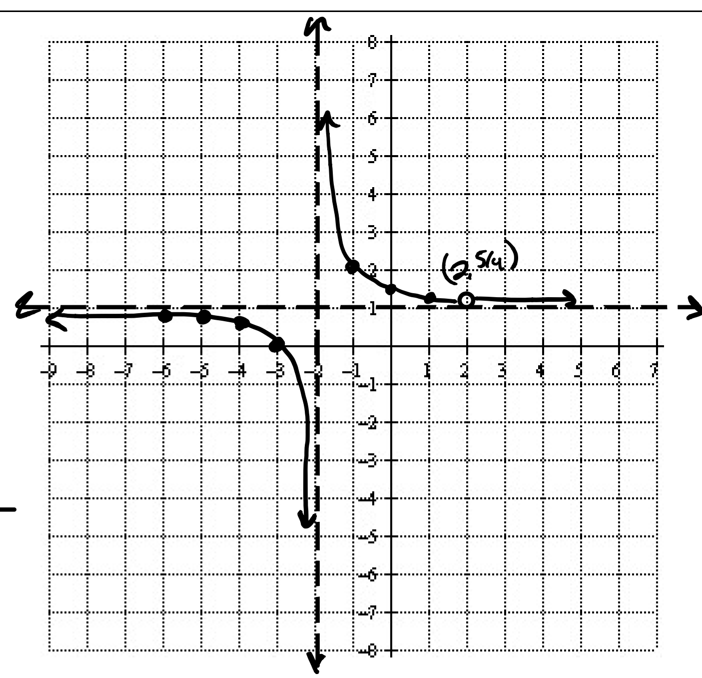
7. Sketch a detailed graph of the function on the grid to the right. You will need to use a minimum of 8 points—4 points on each branch.



x	$f(x) = \frac{x+3}{x+2}$
−4	$\frac{-4+3}{-4+2} = \frac{-1}{-2}$
−5	$\frac{-5+3}{-5+2} = \frac{-2}{-3}$
−6	$\frac{-6+3}{-6+2} = \frac{-3}{-4}$
−1	$\frac{-1+3}{-1+2} = \frac{2}{1}$
1	$\frac{1+3}{1+2} = \frac{4}{3}$

$$\underbrace{3x^2+2x}+\underbrace{3x+2} = x(3x+2)+1(3x+2)$$
$$= (3x+2)(x+1)$$

Use the function $g(x) = \frac{3x^2+5x+2}{x^2+4x+3}$ to answer questions 8 – 14.

8. What is the equation of the function written in completely factored form?

$$g(x) = \frac{(3x+2)(x+1)}{(x+3)(x+1)} \Rightarrow g(x) = \frac{3x+2}{x+3}, \text{ hole at } x=-1$$

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9. If any exist, identify the vertical asymptotes? Explain how you know that they are vertical asymptotes.

g(x) has a noncanceling factor (x+3) in the denominator.

∴ g(x) has a VA at x = −3

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10. Does the function have any holes in the graph? Explain why or why not. What are the coordinates where the hole(s) exist(s)?

g(x) has a factor of (x+1) in both the numerator & denominator.

∴ g(x) has a hole at x = −1

$$g(x) = \frac{3x+2}{x+3} \Rightarrow g(-1) = \frac{3(-1)+2}{(-1)+3} = \frac{-3+2}{2} = \frac{-1}{2}$$

∴ Hole at $(-1, -\frac{1}{2})$

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11. If any exist, identify the horizontal asymptotes. Explain how you know that they are horizontal asymptotes.

g(x)'s numerator and denominator have the same degree

∴ g(x) has a HA at y = 3, which is the ratio of lead coefficients.

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12. What is/are the zero(es) of the function? Show your work.

$3x+2=0$

$3x=-2$

$x=-2/3$

∴ g(x) has a zero at $x = -2/3$

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13. What are the domain and range of the function? Give your answer in interval notation.

Domain $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$

Range $(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},3)\cup(3,\infty)$

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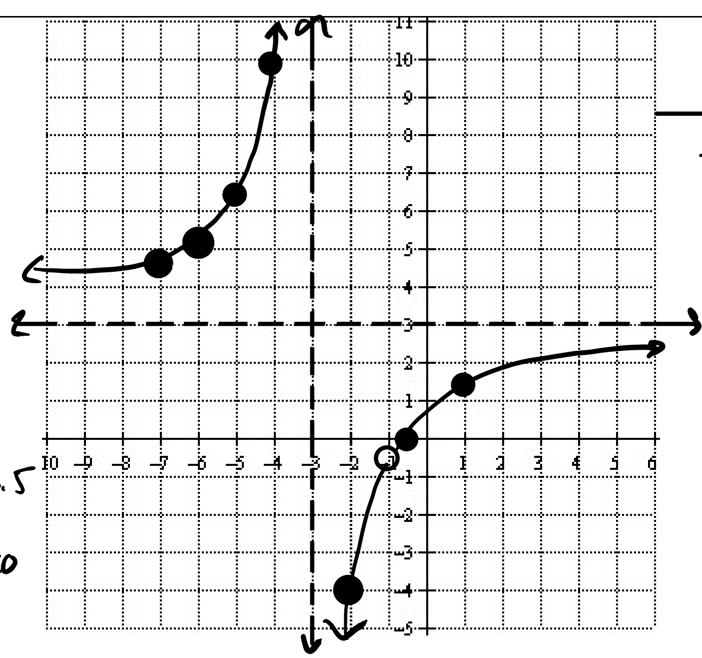
14. Sketch a detailed graph of the function on the grid to the right. You will need to use a minimum of 8 points—4 points on each branch.

X	$\frac{3x+2}{x+3}$
−7	$\frac{3(-7)+2}{-7+3} = \frac{-19}{-4} = 4.75$
−6	$\frac{3(-6)+2}{-6+3} = \frac{-16}{-3} = 5.\bar{3}$
−5	$\frac{3(-5)+2}{-5+3} = \frac{-13}{-2} = 6.5$
−4	$\frac{3(-4)+2}{-4+3} = \frac{-10}{-1} = 10$



X	$\frac{3x+2}{x+3}$
−2	$\frac{3(-2)+2}{-2+3} = \frac{-4}{1}$
1	$\frac{3(1)+2}{1+3} = \frac{5}{4}$

Use the function $g(x) = \frac{2x-6}{x^2-4x+3}$ to answer questions 15 – 21.

15. What is the equation of the function written in completely factored form?

$g(x) = \frac{2(x-3)}{(x-3)(x-1)} \Rightarrow g(x) = \frac{2}{x-1}$, hole $x=3$

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16. If any exist, identify the vertical asymptotes? Explain how you know that they are vertical asymptotes.

g(x) has a noncanceling factor (x-1) in the denominator.
∴ g(x) has a VA at x = 1

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17. Does the function have any holes in the graph? Explain why or why not. What are the coordinates where the hole(s) exist(s)?

g(x) has a canceling factor (x-3)
∴ g(x) has a hole at (3, 1)

$g(3) = \frac{2}{3-1} = \frac{2}{2}$
$g(3) = 1$

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18. If any exist, identify the horizontal asymptotes. Explain how you know that they are horizontal asymptotes.

g(x)'s numerator's degree < the denominator's degree.
∴ g(x) has a horizontal asymptote at y = 0.

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19. What is/are the zero(es) of the function? Show your work.

g(x) has no zeros b/c there is no-noncanceling factor in the numerator.

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20. What are the domain and range of the function? Give your answer in interval notation.

D: $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$
R: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

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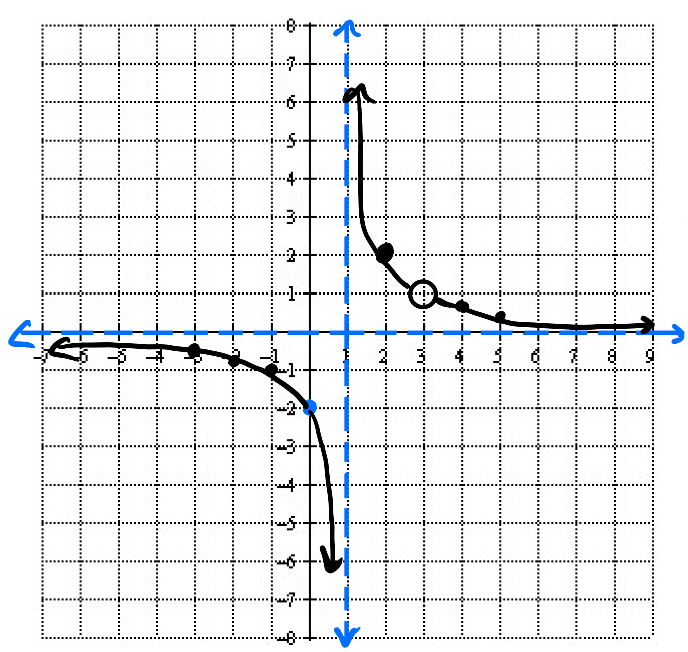
21. Sketch a detailed graph of the function on the grid to the right. You will need to use a minimum of 8 points—4 points on each branch.

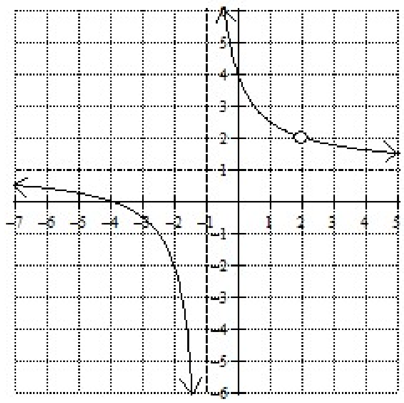
$g(x) = \frac{2}{x-1}$

$g(-3) = \frac{2}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$
$g(-2) = \frac{2}{-2-1} = \frac{2}{-3}$
$g(-1) = \frac{2}{-1-1} = \frac{2}{-2} = -1$



$g(x) = \frac{2}{x-1}$

$g(2) = \frac{2}{2-1} = \frac{2}{1} = 2$
$g(4) = \frac{2}{4-1} = \frac{2}{3}$
$g(5) = \frac{2}{5-1} = \frac{2}{4} = \frac{1}{2}$



The graph of a rational function, $F(x)$, is pictured above. Answer the following questions.

22. What can be said about the degree of the numerator of $F(x)$ compared to the degree of the denominator? Give a reason.

$F(x)$ has a horizontal asymptote at $y=1$

$\therefore$ The degrees of the numerator and denominator are equal.

23. If a is the leading coefficient of the numerator and b is the leading coefficient of the denominator, what is the value of $\frac{a}{b}$? Give a reason.

$F(x)$ has a horizontal asymptote at $y=1$.

$\therefore \frac{a}{b} = 1$

24. What factor(s) is/are guaranteed to be in the denominator of $F(x)$? Give a reason for your answer.

$F(x)$ is discontinuous at $x=-1$ and $x=2$

$\therefore$ $F(x)$ has factors of $(x+1)$ and $(x-2)$ in the denominator.

25. What factor is guaranteed to be in the numerator of $F(x)$ that does not cancel out? Give a reason.

$F(x)$ has a zero at $x=-4$.

$\therefore$ $(x+4)$ is a non-canceling factor of the numerator.

26. What factor is guaranteed to be in both the numerator and denominator of $F(x)$? Give a reason.

$F(x)$ has point discontinuity at $x=2$

$\therefore$ $(x-2)$ is a factor of both the numerator and denominator of $F(x)$

27. What factor is guaranteed to be in the denominator that does not cancel out? Give a reason.

$F(x)$ has a vertical asymptote at $x=-1$.

$\therefore$ $(x+1)$ is a non-canceling factor of the denominator.

28. In both factored and standard form, find an equation of $F(x)$. Give two graphical reasons why your standard form equation makes sense based on the graph.

$$F(x) = \frac{(x+4)(x-2)}{(x+1)(x-2)}$$

$$F(x) = \frac{x^2+2x-8}{x^2-x-2}$$

1. The ratio of the lead coefficients is $\frac{1}{1}=1$. $F(x)$ has a horizontal asymptote at $y=1$.

2. The ratio of the constant terms is $\frac{-8}{-2}=4$ $F(x)$ has a y-int at $(0,4)$