Homework 5.4

For problems 1 – 6, determine the equation of the horizontal asymptote of the function and give a reason. If the function does not have a horizontal asymptote, explain why it does not.

1. $f(x) = \frac{3x^2+10x+8}{x^2-2x-8}$

The degree of the numerator = degree of denominator

$\therefore$ HA @ $y = \frac{3}{1} = 3$

2. $h(x) = \frac{2x-6}{x^2-x-2}$

The degree of the numerator < degree of denominator

$\therefore$ HA @ $y = 0$

3. $p(x) = \frac{2x-3x^2}{2x^2-x-3}$

The degree of the numerator = degree of denominator

$\therefore$ HA @ $y = \frac{-3}{2}$

4. $h(x) = \frac{-2x^2-5x-2}{2x^2-7x+3}$

The degree of the numerator = degree of denominator

$\therefore$ HA @ $y = \frac{-2}{2} = -1$

5. $f(x) = \frac{2x^2+8}{-2x-8}$

The degree of the numerator > degree of denominator

$\therefore$ NO HA

6. $P(x) = \frac{(3-2x)(x+3)}{(2+x)(3-x)} = \frac{-2x^2+\cdots}{-x^2+\cdots}$

The degree of the numerator = degree of denominator

$\therefore$ HA @ $y = \frac{-2}{-1} = 2$

For problems 7 and 8, find the equation of the slant asymptote of the function. If the function does not have a slant asymptote, explain why it does not.

7. $f(x) = \frac{x^2+5x+8}{x+3}$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 8 \\ & 0 & -3 & -6 \\ \hline & 1 & 2 & \boxed{R\,2} \end{array}$$

$\therefore$ SA @ $y = x + 2$

8. $f(x) = \frac{x^2-x+1}{2x-2}$

CAREFUL

$f(x) = \frac{x^2-x+1}{2(x-1)}$

$$\begin{array}{r|rrr} 1 & 1 & -1 & 1 \\ & 0 & 1 & 0 \\ \hline & \frac{1}{2} & \frac{0}{2} & \boxed{R\,1} \end{array}$$

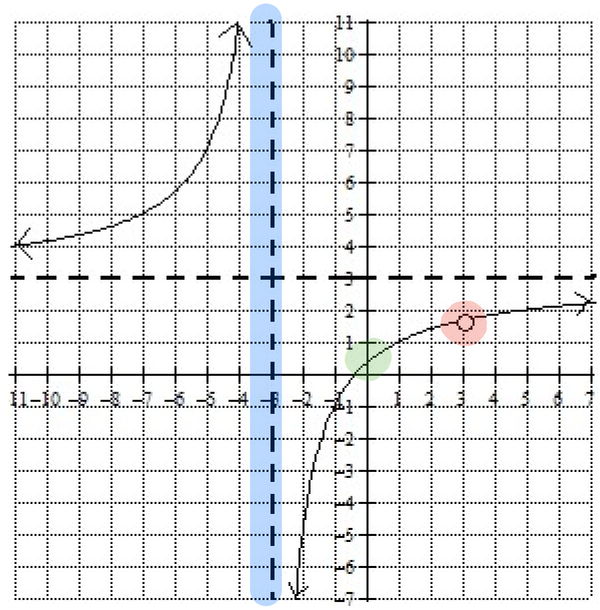
$\therefore$ SA $y = \frac{1}{2}x$

Answer questions 9 – 11 using the graph of the rational function pictured to the right. The graph of the function has a removable point discontinuity at $\left(3, \frac{5}{3}\right)$ and an x – intercept at $\left(-\frac{1}{3},\ 0\right)$.

9. What can be said about the degree of the numerator of the equation of $f(x)$ compared to the degree of the denominator? Give a reason for your answer.

$f(x)$ has a HA, but not at $x=0$.

$\therefore$ The degree of the numerator and denominator are equal.



10. In factored and standard form, what is the equation of $f(x)$?

$$f(x) = \frac{(3x+1)(x-3)}{(x+3)(x-3)}$$

$$f(x) = \frac{3x^2-8x-3}{x^2-9}$$

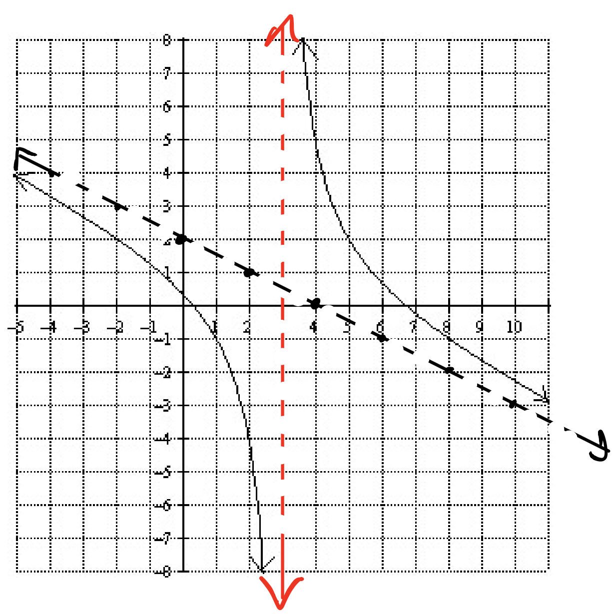
11. Give two reasons why the standard form equation you wrote in exercise 10 makes sense in the context of the graph.

- The ratio of the constants of $f(x)$ is $\frac{-3}{-9} = \frac{1}{3}$ which is the y-int of the graph.
- The ratio of the lead coefficients is $\frac{3}{1} = 3$ and the HA is $y=3$.

12. The graph of the function $h(x) = \frac{x^2-7x+2}{-2x+6}$ is pictured to the right. Find the equation(s) of the asymptotes of the graph of $h(x)$ and draw them on the graph.

$$h(x) = \frac{x^2-7x+2}{-2(x-3)}$$

Careful!



VA

$x=3$

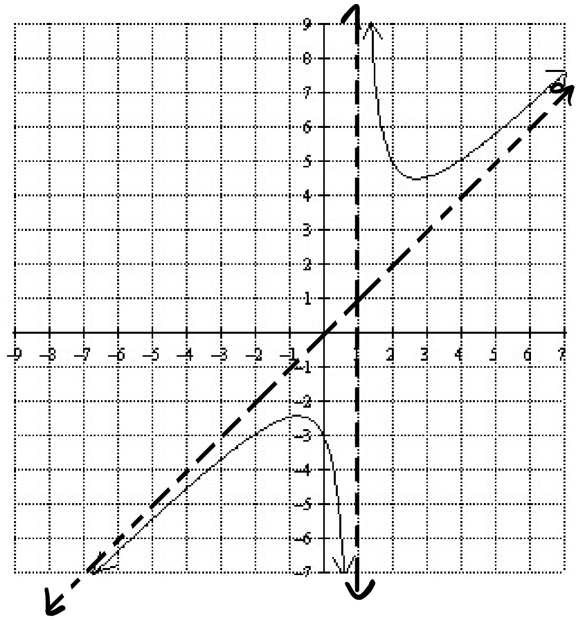
SA

$$\begin{array}{r|rrr} 3 & 1 & -7 & 2 \\ & 0 & 3 & -12 \\ \hline & 1 & -4 & \boxed{R} \\ & \overline{-2} & \overline{-2} & \end{array}$$

Vertical Asymptote at $x=3$

Slant Asymptote at $y=-\frac{1}{2}x+2$

13. The graph of the function $g(x) = \frac{x^2-x+3}{x-1}$ is pictured to the right. Find the equation(s) of the asymptotes of the graph of $h(x)$ and draw them on the graph.



VA

$x=1$

SA

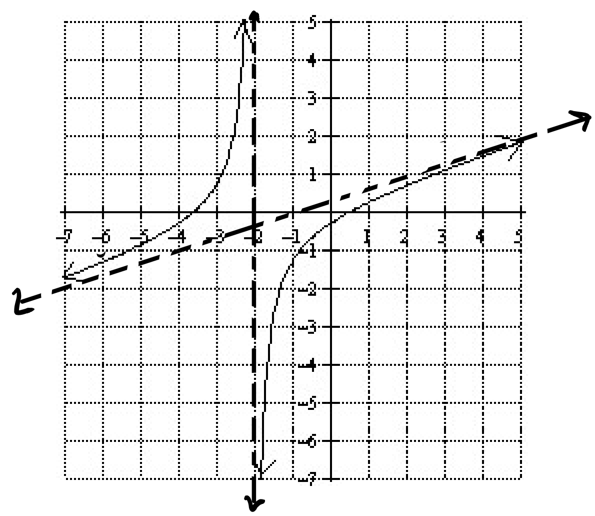
$$\begin{array}{r|rrr} 1 & 1 & -1 & 3 \\ & 0 & 1 & 0 \\ \hline & 1 & 0 & \boxed{R\,3} \end{array}$$

Vertical Asymptote at $x=1$

Slant Asymptote at $y=x$

14. The graph of the function $h(x) = \frac{x^2+3x-2}{3x+6}$ is pictured to the right. Find the equation(s) of the asymptotes of the graph of $h(x)$ and draw them on the graph.

$h(x) = \frac{x^2+3x-2}{3(x+2)}$ Careful!



VA

$x=-2$

SA

$$\begin{array}{r|rrr} -2 & 1 & 3 & -2 \\ & 0 & -2 & -2 \\ \hline & \frac{1}{3} & \frac{1}{3} & \boxed{R\,-4} \end{array}$$

Vertical Asymptote at $x=-2$

Slant Asymptote at $y=\frac{1}{3}x+\frac{1}{3}$