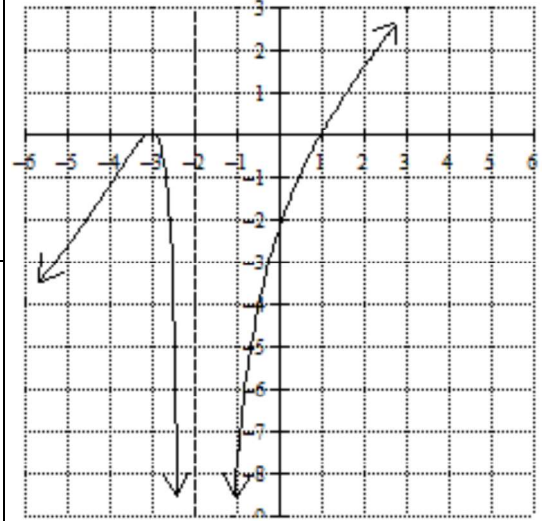## Homework 5.1

The graph of the rational function $h(x) = \frac{(x+3)^2(x-1)}{(x+2)^2}$ is shown to the right. Graphically determine the solutions to the following inequalities. Give a reason for your solution intervals based on the graph.

1. $h(x) < 0$

$h(x) < 0$ on $(-\infty, -3) \cup (-3, -2) \cup (-2, 1)$

b/c $h(x)$ is below the x-axis on these intervals.

2. $h(x) \geq 0$

$h(x) \geq 0$ at $x = -3$ and $[1, \infty)$

b/c $h(x)$ is above or on the x-axis on these x-values.

For exercises 4 and 5 below, give a reason for your solution based on your sign analysis performed in exercise 3 below.

3. Perform a sign analysis for the function $h(x) = \frac{(x+3)^2(x-1)}{(x+2)^2}$ that will be used to solve the Inequalities in exercises 4 and 5 below.
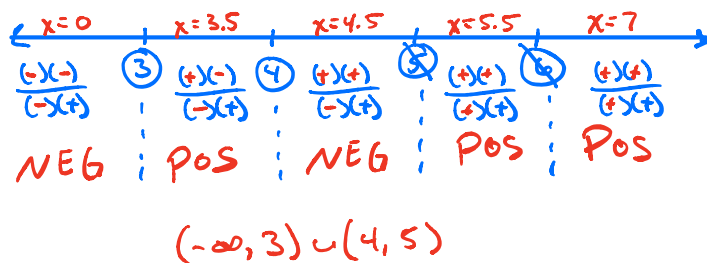
$$x = -4 \qquad x = -2.5 \qquad x = 0 \qquad x = 2$$

$$\frac{(+)(-)}{+} \overset{-3}{\bigcirc} \qquad \frac{(+)(-)}{+} \overset{-2}{\oslash} \qquad \frac{(+)(-)}{+} \overset{1}{\bigcirc} \qquad \frac{(+)(+)}{+}$$

$$NEG \qquad NEG \qquad NEG \qquad POS$$

4. $\frac{(x+3)^2(x-1)}{(x+2)^2} < 0$

$h(x) < 0$ on $(-\infty, -3) \cup (-3, -2) \cup (-2, 1)$
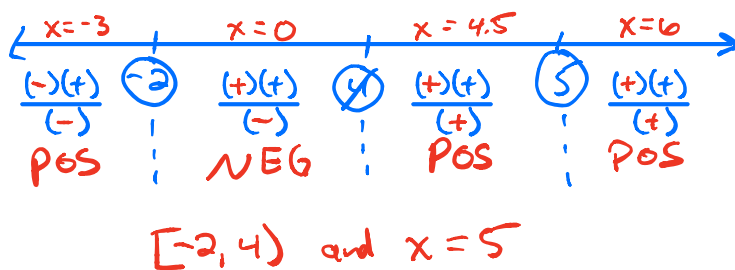
b/c $h(x)$ is negative on these intervals.

5. $\frac{(x+3)^2(x-1)}{(x+2)^2} \geq 0$

$h(x) \geq 0$ at $x = -3$ and $[1, \infty)$

b/c $h(x)$ is zero or positive on these x-values.

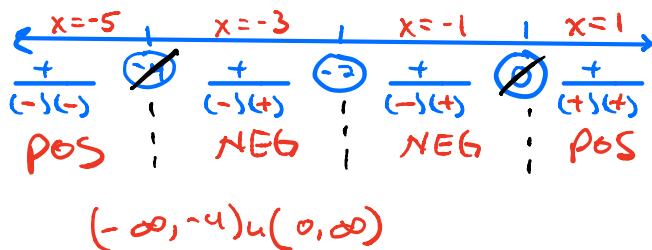Algebraically solve each of the following rational inequalities. Show your sign analysis.

6. $\frac{(x-3)(x-4)}{(x-5)(x-6)^2} < 0$

$x=0$ $x=3.5$ $x=4.5$ $x=5.5$ $x=7$

$\frac{(-)(-)}{(-)(+)}$ ③ $\frac{(+)(-)}{(-)(+)}$ ④ $\frac{(+)(+)}{(-)(+)}$ ⑤ $\frac{(+)(+)}{(+)(+)}$ ⑥ $\frac{(+)(+)}{(+)(+)}$

NEG   POS   NEG   POS   POS

$(-\infty, 3) \cup (4, 5)$

7. $\frac{(x+2)(x-5)^2}{(x-4)} \le 0$

$x=-3$ $x=0$ $x=4.5$ $x=6$

$\frac{(-)(+)}{(-)}$ ⊝2 $\frac{(+)(+)}{(-)}$ ④ $\frac{(+)(+)}{(+)}$ ⑤ $\frac{(+)(+)}{(+)}$

POS   NEG   POS   POS

$[-2, 4)$ and $x=5$

8. $\frac{x^2+4x+4}{x^2+4x} > 0$ $\implies$ $\frac{(x+2)^2}{x(x+4)} > 0$

$x=-5$ $x=-3$ $x=-1$ $x=1$

$\frac{+}{(-)(-)}$ ⊘ $\frac{+}{(-)(+)}$ ⊝2 $\frac{+}{(-)(+)}$ ⊘ $\frac{+}{(+)(+)}$

POS   NEG   NEG   POS

$(-\infty, -4) \cup (0, \infty)$

9. $\frac{4}{x-3} \ge \frac{2}{x-5}$

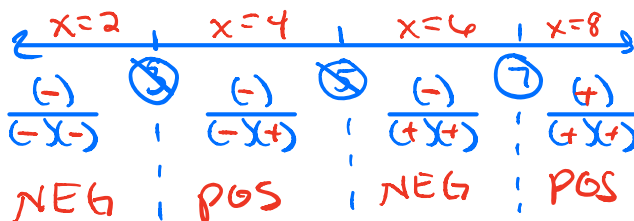$\frac{(x-5)}{(x-5)} \frac{4}{x-3} - \frac{2}{x-5} \frac{(x-3)}{(x-3)} \ge 0$

$\frac{4x-20}{(x-5)(x-3)} - \frac{2x-6}{(x-5)(x-3)} \ge 0$

$\frac{4x-20-2x+6}{(x-5)(x-3)} \ge 0$

$\frac{2x-14}{(x-5)(x-3)} \ge 0$

$\frac{2(x-7)}{(x-5)(x-3)} \ge 0$

$x=2$ $x=4$ $x=6$ $x=8$

$\frac{(-)}{(-)(-)}$ ③ $\frac{(-)}{(-)(+)}$ ⑤ $\frac{(-)}{(+)(+)}$ ⑦ $\frac{(+)}{(+)(+)}$

NEG   POS   NEG   POS

$(3, 5) \cup [7, \infty)$