Review FRQ

FREE RESPONSE

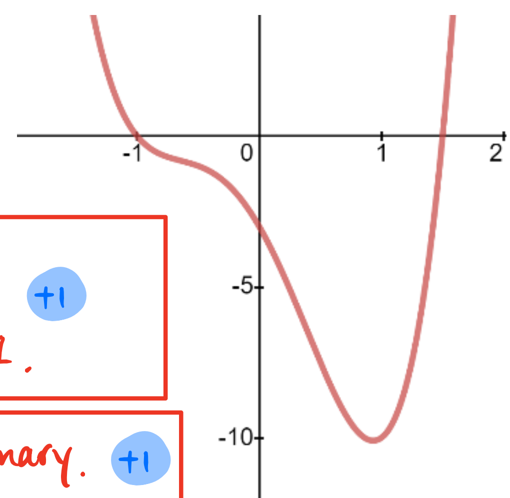
FRQ 1: Consider the function $h(x) = 4x^4 + 2x^3 - 6x^2 - 7x - 3$ to answer the following questions.

a. Find an equation for $h(-x)$. Specifically explain what possibility about the roots of $h(x)$ can be determined from this equation.

$\boxed{h(-x) = 4x^4 - 2x^3 - 6x^2 + 7x - 3}$ +1

h(-x) has 3 sign changes. By Decarte's Rule of Signs, h(x) has 3 or 1 negative roots. +1

b. Use the graph of the function $h(x)$. Then, determine how many of the roots are imaginary. Give a reason for your answer.



By the FTA, h(x) has 4 roots b/c h(x) is quartic. The graph of h(x) shows 2 roots each with multiplicity of 1. +1

∴ The 2 remaining roots are imaginary. +1

c. Make a complete list of the rational roots that are possible for $h(x)$. Then, after comparing the list to the roots indicated in the graph, choose the two most probable rational roots.

Possible Rational Roots: $\frac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$ +1

Two Most Probable Roots: $-1, \frac{3}{2}$ +1

d. Find all of the roots, real and/or imaginary, of $h(x)$. Show all of your work and leave your answers in simplified, exact, complex form, if necessary.

$\boxed{-1}$	4	2	-6	-7	-3
	0	-4	2	4	3
$\boxed{3/2}$	4	-2	-4	-3	$\underline{0}$
	0	6	6	3	
	4	4	2	$\underline{0}$	

+1

$h(x) = (x+1)(x-3/2)(4x^2+4x+2)$

$h(x) = (x+1)(x-3/2)2(2x^2+2x+1)$

$2x^2+2x+1=0$

$DISC = b^2-4ac$

$= (2)^2-4(2)(1)$

$= 4-8$

$DISC = -4$

$x = \frac{-b \pm \sqrt{DISC}}{2a}$

$= \frac{-2 \pm \sqrt{-4}}{2(2)}$

$= \frac{-2 \pm 2i}{4}$

$x = \frac{-1 \pm i}{2}$

+1

Roots of $h(x)$: $-1, 3/2, \frac{-1-i}{2}, \frac{-1+i}{2}$ +1

Calculator NOT Permitted

zero

x	-3	-2	0	1	3	4
$F(x)$	50	16	-4	-2	-4	-20

FRQ1 : The table above shows function values of a cubic polynomial function, $F(x)$. The function has two distinct zeros, $x = a$ and $x = b$, such that $a < 0$ and $b > 0$. Additionally, one of the zeros has a multiplicity of two.

a. Determine the left and right hand behavior of $F(x)$ based on the table of values. Give a reason for your answers.

Not great reasons

when $x=-3$, $F(x)=50$.

$\therefore \lim_{x \to -\infty} F(x) = \infty$

+1

when $x=4$, $F(x)=-20$.

$\therefore \lim_{x \to \infty} F(x) = -\infty$

+1

b. What can be said about the leading coefficient of $F(x)$? Justify your reasoning.

- The graph of $F(x)$ falls to the right. +1

$\therefore$ The leading coefficient is negative. +1

c. Between what two *x* – values in the table does the zero *x* = *a* lie? What is its multiplicity? Justify your reasoning.

- $f(-2) = 16$ and $f(0) = -4$

$\therefore$ $f(x)$ changes from + to − between $x = -2$ and $x = 0$.

$\therefore$ $f(x)$ crosses the x-axis between $x = -2$ and $x = 0$. +1 Wording can vary greatly

$\therefore$ $-2 < a < 0$ with "a" having ODD multiplicity of 1 +1

b/c $f(x)$ has two zeros so the odd multiplicity < 3 +1

d. Between what two *x* – values in the table does the zero *x* = *b* lie? What is its multiplicity? Justify your reasoning.

- $f(x) \le 0$ on $(0, \infty)$ +1
- $f(x)$ is increasing on interval $(0,1)$ and decreasing on $(3, \infty)$

$\therefore$ $f(x)$ is tangent to the x-axis at $x = b$ with even multiplicity 2 where $0 < b < 3$. +1

Calculator NOT Permitted

$x = 2i$

$(x-3)^2$ ... FACTOR Theorem

FRQ 2: A function, $g(x)$, has a root of $x = 2i$ and a root of $x = 3$, which has a multiplicity of 2.

a. Find an equation of $g(x)$.

Complex Conjugate root theorem

$x = \pm 2i$

$x^2 = -4$

$x^2 + 4 = 0$

FACTOR Theorem

$(x-3)^2 = x^2 - 6x + 9$

$g(x) = (x^2+4)(x^2-6x+9)$ +1 +1

$= x^4 - 6x^3 + 9x^2 + 4x^2 - 24x + 36$

$g(x) = x^4 - 6x^3 + 13x^2 - 24x + 36$ +1

b. Determine the left- and right-hand behavior of $g(x)$. Justify your reasoning.

$g(x)$ is even degree with lead coefficient positive. +1

$\therefore \lim_{x\to-\infty} g(x) = \infty \qquad \lim_{x\to\infty} g(x) = \infty$

+1

c. A quartic function in the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ is such that the coefficients of the quadratic and linear terms are 10 and –18, respectively. Additionally, $f(0) = 9$ and $x = 1$ is a root of multiplicity of 2. What is the value of $(a + b)$?

$-18x$

$10x^2$

- $f(0)=9 \;\therefore\; e=9 \implies f(x)=ax^4+bx^3+10x^2-18x+9$ +1

- $x=1$ is a root, $\therefore$ so $f(1)=0$

$f(1) = a(1)^4 + b(1)^3 + 10(1)^2 - 18(1) + 9$ +1

$0 = a + b + 10 - 18 + 9$

$0 = a+b+1$

$a+b=-1$ +1

Review

MULTIPLE CHOICE – Calculator Permitted

3 roots

1. It is known that a polynomial function, $f(x)$, has roots of $x = 2$, which has multiplicity of 3, and $x = 2 - i$. Minimally, what type of polynomial function is $f(x)$?

2 roots

$2 + 3 = 5$

   A. quadratic

   B. cubic

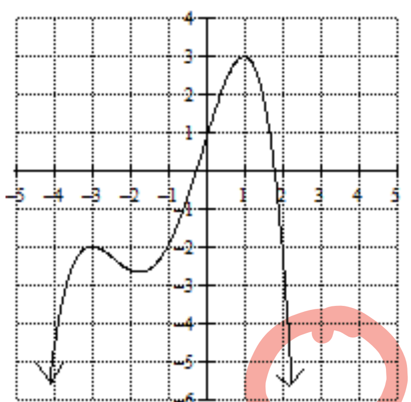
   C. quartic

   D. quintic

   E. linear

2. Which of the following statements is true about the graph of $P(x)$ pictured to the right?

Even Deg ≥ 4



$\lim_{x \to \infty} P(x) = -\infty$

$\therefore$ neg Lead coeff

   A. $P(x)$ is a quadratic function whose equation has a negative leading coefficient.

   B. $P(x)$ is a quadratic function whose equation has a positive leading coefficient.

   C. $P(x)$ is a quartic function whose equation has a negative leading coefficient.

   D. $P(x)$ is a quartic function whose equation has a positive leading coefficient.

   E. $P(x)$ is a cubic function whose equation has a negative leading coefficient.

$x^2(-3x^3-3x+2)$

3. If $f(x) = -3x^5 - 3x^3 + 2x^2$, which of the following statements is true?

   A. $x = 0$ is not a root of $f(x)$.

   B. $x = 0$ is a root of $f(x)$ 1 time.

   C. $x = 0$ is a root of $f(x)$ 2 times.

   D. $x = 0$ is a root of $f(x)$ 3 times.

   E. $x = 0$ is a root of $f(x)$ 4 times.

4. Which of the following statements is/are true about the polynomial function, $P(x)$?

$$P(x) = -3x^5 - 2x^4 + 2x^2 - x + 2$$

I.	$\lim_{x\to\infty} P(x) = -\infty$ True
II.	All of the possible rational roots of $P(x)$ are $\pm 1,\ \pm 2,\ \pm\frac{1}{3}$. False PRR $= \dfrac{\pm 1, \pm 2}{\pm 1, \pm 3}$
III.	There can be either 3 or 1 positive root(s) of $P(x)$. True

$p(x) = - \underbrace{-\ +}_{1} \underbrace{\ -}_{2} \underbrace{\ +}_{3}$

   A. I only

   B. II and III only

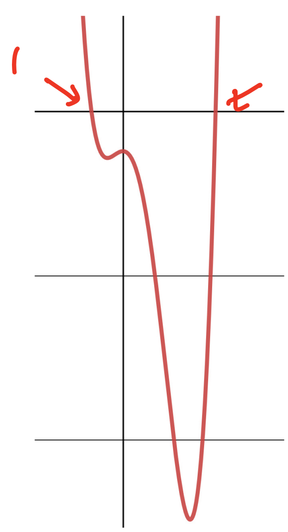
   C. I and II only

   D. I, II, and III

   E. I and III only

5. Which of the following is the correct combination of the types of roots for the function $g(x) = x^4 - 4x^3 - 7x^2 - 12$?

	Positive	Negative	Zero	Imaginary
A.	3	1	0	0
B.	1	3	0	0
C.	2	2	0	0
D.	1	1	0	2
E.	0	0	0	4



6. The graph of the function $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ is pictured. Which of the following is true?

$a>0$

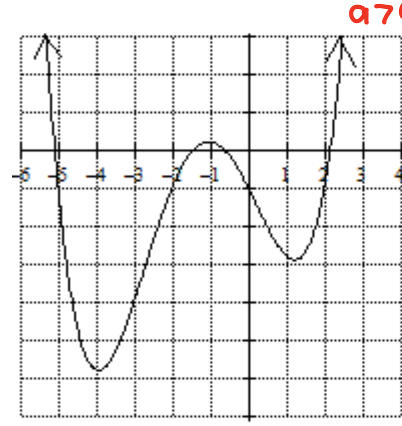
A. The value of $a > 0$.

B. The value of $a < 0$.

C. The value of $e = 1$.

D. Both A and C

E. Both B and C.



$e=-1$

7. At which of the following values of x does the graph of $h(x) = -2x^4 - 5x^3 + 4x^2 + 12x$ have an approximate point of inflection?

$x = \frac{-2+-.806}{2}$

$x \approx \frac{-.806+.931}{2}$ I. $x = 0.063$ II. $x = -1.403$ III. $x = -2$?

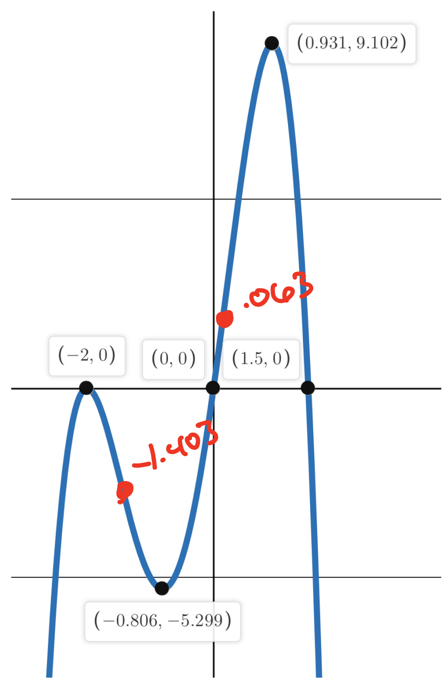
A. II and III only

B. I only

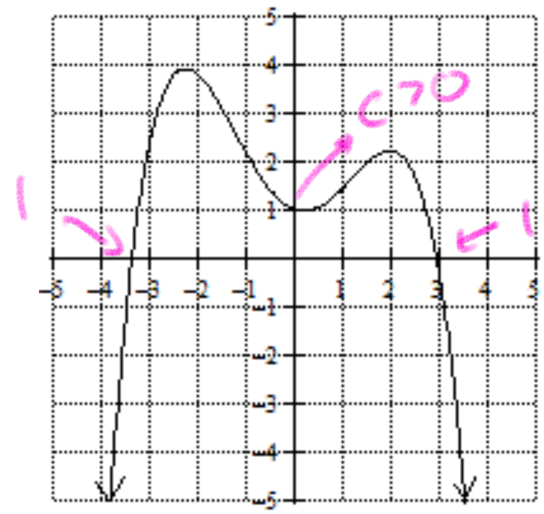
C. I and II only

D. I, II, and III only

E. Cannot be determined



The graph of a quartic function, $p(x)$, is pictured. Use the graph for questions 8 and 9.



Even
Degree ≥ 4

8. Which of the following conclusions can be made about $p(x)$.

   A. The equation of $p(x)$ has an even number of sign changes.

   B. The equation of $p(-x)$ has an odd number of sign changes.

   C. The constant term, c, of $p(x)$ is such that $c > 0$.

   D. Both A and C are true.

   E. Both B and C are true.

9. Which of the following can be concluded about the roots of $p(x)$?

   A. $p(x)$ has one irrational root, one rational root, and two imaginary roots.

   B. $p(x)$ has two real roots and two imaginary roots. likely

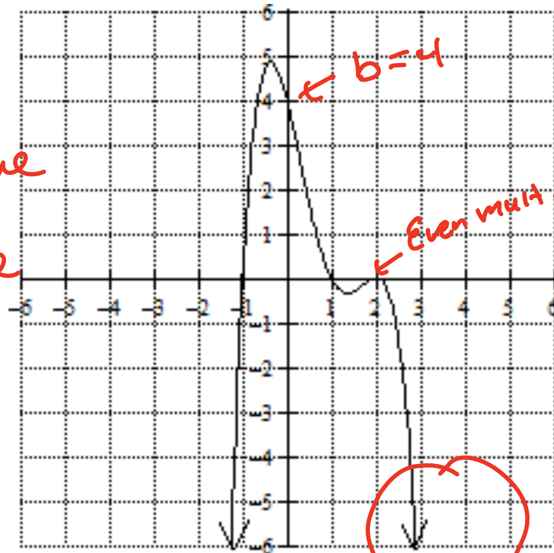
   C. $p(x)$ has four imaginary roots.

   D. $p(x)$ has four real roots.

   E. None of these conclusions can be reached about $p(x)$.

MULTIPLE CHOICE – Calculator Permitted Section

10. Pictured to the right is the graph of the function $g(x) = ax^4 + 4x^3 - 3x^2 - 4x + b$. Which of the following statements is/are true?



$\leftarrow b=4$

$\leftarrow$ Even mult 2

$a < 0$

I. The value of $a > 0$. False

II. The factor $(x - 2)$ is a factor of $g(x)$ twice. True

III. The value of b in the equation is 4. True

A. II only

B. I and II only

C. I and III only

D. II and III only

E. III only

11. Find all of the roots, real and/or imaginary, of the function $f(x) = x^3 + 6x^2 + 12x + 7$.

A. $x = -1, 7$

B. $x = -1, \frac{-5 \pm i\sqrt{3}}{2}$

C. $x = -1, \frac{-5 \pm 3i\sqrt{2}}{2}$

D. $x = -1, \frac{-5 \pm \sqrt{3}}{2}$

E. Roots cannot be determined

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 12 & 7 \\ & & -1 & -5 & -7 \\ \hline & 1 & 5 & 7 & 0 \end{array}$$

$x^2 + 5x + 7 = 0$

$x = \frac{-b \pm \sqrt{DSC}}{2a}$

$DSC = b^2 - 4ac$
$= 5^2 - 4(1)(7)$
$= 25 - 28$
$= -3$

$x = \frac{-5 \pm \sqrt{-3}}{2(1)}$

12. Which of the following correctly describes the number of negative roots <u>possible</u> of the function $h(x) = 2x^4 + 3x^3 + 2x^2 - 2x - 3$ according to Descartes' Rule of Signs?

$h(-x) = 2x^4 \underbrace{-\,3x^3}_{1} \underbrace{+\,2x^2}_{2} + \underbrace{2x - 3}_{3}$

A. 3 or 1 B. 2 or 0 C. Only 1

D. 4, 2, or 0 E. Only 2

And $-2i$

13. A quartic function has roots of $x = 1$, -3, and $2i$. What is the equation of $f(x)$?

A. $f(x) = x^4 - 2x^3 + x^2 - 8x - 12$

B. $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

C. $f(x) = x^3 - 2x + 3ix - 3$

D. $f(x) = x^4 - 2x^3 - 7x^2 + 8x - 12$

E E. $f(x) = x^4 + 2x^3 + x^2 + 8x - 12$

$x = \pm 2i$

$x^2 = -4$

$x^2 + 4 = 0$

$f(x) = (x-1)(x+3)(x^2+4)$

$= (x^2 + 2x - 3)(x^2 + 4)$

$= x^4 + 2x^3 - 3x^2 + 4x^2 + 8x - 12$

$f(x) = x^4 + 2x^3 + x^2 + 8x - 12$

$k = 0$

14. Find the value of k so that the binomial $(x + 3)$ is a factor of the function $f(x) = x^3 + kx^2 - 5x - 6$.

A. 2

B. -3

C. -2

D. $\frac{2}{3}$

E. None of these

$$\begin{array}{r|cccc} -3 & 1 & k & -5 & -6 \\ & & -3 & (-3k+9) & (9k-12) \\ \hline & 1 & (k-3) & (-3k+4) & 9k-18 \end{array}$$

$9k - 18 = 0$

$9k = 18$

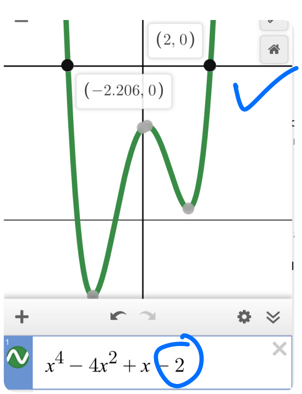
$k = 2$

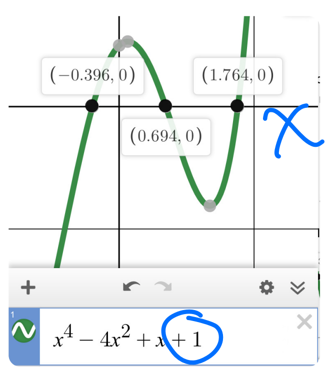
15. For which of the following values of k does the function $g(x) = x^4 - 4x^2 + x + k$ have exactly two distinct real roots?

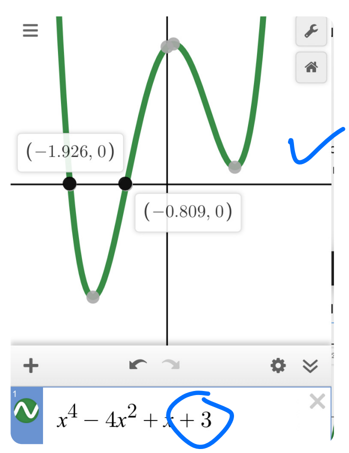
I. $k = -2$ II. $k = 1$ III. $k = 3$

A. I and III only B. II only C. III only

D. II and III only E. I only







16. The table of values below represents a cubic polynomial function, $F(x) = ax^3 + 2x^2 - 5x + b$, that has two negative roots and one positive root. Which of the following statements is/are true?

(x+3) (x+1)

x	−5	−3	−2	−1	0	1	3	5
$F(x)$	−56	0	4	0	−6	−8	24	144

$b=-6$ → $a>0$

I. The value of $a < 0$ and $b = -6$. False

II. In factored form, the equation of $F(x)$ would contain the factor $(x + 3)$. ✓

III. The graph of $F(x)$ is ~~tangent~~ crosses to the x – axis at $x = -1$. X

A. I, II and III

B. I only

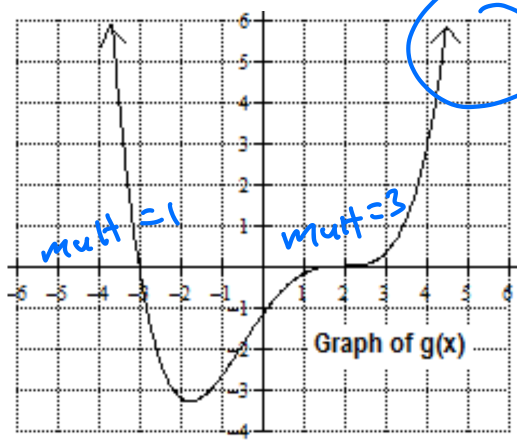
C. I and II only

D. II only

E. II and III only

MULTIPLE CHOICE – Calculator NOT Permitted Section

17. Which of the following statements is/are true about the quartic function, $g(x)$, pictured?



$a > 0$

ODD ≥ 3

I. The graph has one positive root that has a multiplicity of 2. False

II. The leading coefficient of the equation of $g(x)$ is positive. True

III. All of the roots of $g(x)$ are real. True

A. I only
B. II only
C. I and III only
D. II and III only
E. I and II only

18. If $x = 3$ is one root of the function $f(x) = x^3 - x^2 - 4x - 6$, what are the other two roots?

A. $x = 1 + i$ and $x = 1 - i$

B. $x = 1$ and -2

C. $x = -1 + i$ and $x = -1 - i$

D. $x = -1$ and -2

E. $x = -1 + 2i$ and $x = -1 - 2i$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -4 & -6 \\ & & 3 & 6 & 6 \\ \hline & 1 & 2 & 2 & \boxed{0} \end{array}$$

$x^2 + 2x + 2 = 0$

Complete square

$x^2 + 2x + 1 = -2 + 1$

$(x+1)^2 = -1$

$x + 1 = \pm i$

$x = -1 \pm i$

19. The synthetic division of a polynomial function, $g(x)$ is shown to the right. Which of the following conclusions can be made?

$g(2) = -2$ constant

2	−2	0	3	8
	0	−4	−8	−10
	−2	−4	−5	$\boxed{-2}$

Deg = 3

I. $g(x)$ is a quartic function. False

II. The graph of $g(x)$ is below the x – axis at $x = 2$. True

III. The graph of $g(x)$ crosses the y – axis at (0, 8). True

A. I and III only B. III only C. I, II, and III

D. II only E. II and III only

20. Which of the following is NOT a possible rational root of $g(x) = -2x^3 + 4x^2 - 2x - 6$

A. $-{}^{2}/_{3}$ B. −3 C. 6 D. $-{}^{3}/_{2}$ E. −2

$$PRR = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$$

21. Which of the following could be the complete chart of possible types and numbers of the roots of the function $F(x) = -2x^5 + 3x^3 + 2x^2 - x - 3$?

$F(x) = - \; + \; + \; - \; -$ 1 2

A.

Positive	Negative	Imaginary
2	2	1
2	0	3
0	2	3
0	0	5

B.

Positive	Negative	Imaginary
2	3	0
2	1	2
0	3	2

C.

Positive	Negative	Imaginary
2	3	0
2	1	2

D. D

Positive	Negative	Imaginary
2	3	0
2	1	2
0	3	2
0	1	4

$F(-x) = 2x^5 - 3x^3 + 2x^2 + x - 3$ 1 2 3

more complete

22. Which of the following statements is/are true about the function $f(x) = 2x^3 - 4x^2 + 10x - 12$?

I. The graph will fall to the left and rise to the right. True

II. There is a guaranteed zero on the interval $-1 < x < 1$.

III. One zero of the function is $x = -3$. X

$-3 \,|\, 2 \quad -4 \quad 10 \quad -12$
$\quad -6 \quad 30 \quad -126$
$2 \quad -10 \quad 40 \quad \boxed{-132}$

A. I and II only B. II and III only C. I only

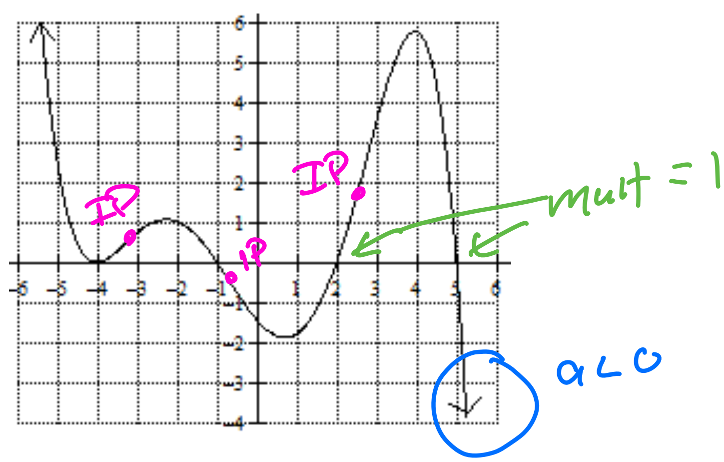
D. III only E. I, II, and III

$1 \,|\, 2 \quad -4 \quad 10 \quad -12$
$\quad 2 \quad -2 \quad 8$
$2 \quad -2 \quad 8 \quad \boxed{-4}$
$\therefore f(1) = -4$

$-1 \,|\, 2 \quad -4 \quad 10 \quad -12$
$\quad -2 \quad 6 \quad -16$
$2 \quad -2 \quad 12 \quad \boxed{-24}$
$\therefore f(-1) = -24$

f(1) & f(-1) are same sign. Can't guarantee a zero between them.

23. Assuming that the function graphed below has no imaginary roots, which of the following statements is/are true about the function?



I. The leading coefficient is negative. True

II. The graph of the function will have 3 points of inflection. True

III. The function has two roots that are positive, one of which has a multiplicity of 2. False

A. I and II only B. III only C. II and III only

D. I and III only E. I, II, and III