Homework 4.4 Calculator Permitted

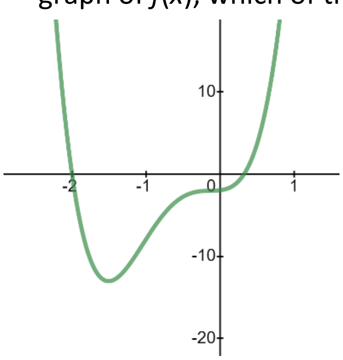
FRQ 1. Consider the function $f(x) = 9x^4 + 21x^3 + 7x^2 + x - 2$ to answer the following questions.

a. Find $f(-2.5)$ and $f(-1.5)$.
What do these values suggest about the graph of $f(x)$ on the interval $-2.5 < x < -1.5$?

$f(-2.5) = 62.688$
$f(-1.5) = -13.06$
+1

$f(-2.5) > 0$ and $f(-1.5) < 0$.
∴ $f(x)$ changes signs between $x = -2.5$ and $x = -1.5$
∴ $f(x)$ has a root between $x = -2.5$ and $x = -1.5$
+1

b. Use Descartes' Rule of Signs to determine the number of possible positive, negative, zero, and imaginary roots of $f(x)$. Make a chart that summarizes your results. Then, after investigating the graph of $f(x)$, which of the combinations from the table is correct and explain why.



$f(x) = 9x^4 + 21x^3 + 7x^2 + x - 2$
$f(-x) = 9x^4 - 21x^3 + 7x^2 - x - 2$

+	= 1
−	= 3 or 1
zero	= 0
i	= 4, 2, 0

+1

P	N	Zero	i
1	3	0	0
1	1	0	2

+1

By the FTA, $f(x)$ has 4 roots.
- $f(x)$ crosses the x-axis once (w/o changing concavity) on both the negative and positive x-axis
- $f(x)$ does not contain the origin

∴ $f(x)$ has 1 pos, 1 neg, 0 zero, and 2 imaginary roots.
+1

c. What are all of the possible rational roots of $f(x)$? Of these possible roots, which two appear to be the most likely possible roots?

$$PRR = \frac{\pm 1, \pm 2}{\pm 1, \pm 3, \pm 9}$$

$$HPRR = -2, \tfrac{1}{3}$$

✓ ✓

+1

d. Find the roots of $f(x)$, real and/or imaginary. Show all of your work.

+1

-2	9	21	7	1	-2
	0	-18	-6	-2	2
$\frac{1}{3}$	9	3	1	-1	0
	0	3	2	1	
	9	6	3	0	

$f(x) = (x+2)(x-\frac{1}{3})(9x^2+6x+3)$

$f(x) = 3(x+2)(x-\frac{1}{3})(3x^2+2x+1)$

$3x^2+2x+1=0$

$$\boxed{\begin{aligned} Disc &= b^2-4ac \\ &= (2)^2-4(3)(1) \\ &= 4-12 \\ Disc &= -8 \end{aligned}}$$

$x = \dfrac{-b \pm \sqrt{Disc}}{2a}$

$x = \dfrac{-2 \pm \sqrt{-8}}{2(3)}$

$x = \dfrac{-2 \pm 2i\sqrt{2}}{6}$

$x = \dfrac{-1 \pm i\sqrt{2}}{3}$ +1

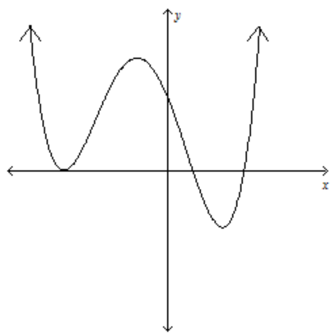
+1

Roots of $f(x)$: $x=-2$, $x=\frac{1}{3}$, $x=\frac{-1-i\sqrt{2}}{3}$, $x=\frac{-1+i\sqrt{2}}{3}$

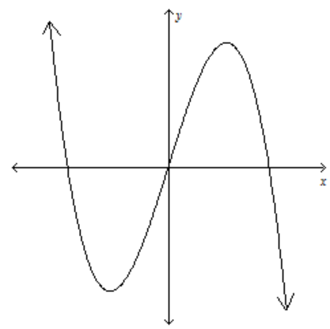
Calculator NOT Permitted

FRQ 2. Pictured below are graphs of two different polynomial functions. All of the zeros of each function are real—none are imaginary. Answer the questions that follow about the two graphs, $f(x)$ and $g(x)$.

Graph of $f(x)$



Graph of $g(x)$



a. Based on the graphs, what types of polynomial functions are $f(x)$ and $g(x)$? Explain your reasoning.

+1 $f(x)$ and $g(x)$ have no imaginary roots. By the FTA, the degree of $f(x)$ and $g(x)$ will be the sum of the multiplicities of the roots.

+1
- $f(x)$ is tangent to the negative x-axis. $\therefore f(x)$ has a root multiplicity ≥ 2.
- $f(x)$ crosses the x-axis twice w/o changing concavity $\therefore f(x)$ has two roots of mult $= 1$

+1/2 $\therefore f(x)$ is even degree ≥ 4.

$g(x)$ crosses the x-axis 3 times w/o changing concavity. $\therefore g(x)$ has 3 roots of mult $= 1$. +1

$\therefore g(x)$ is cubic. +1/2

b. What can be concluded about the value of a, if a is the leading coefficient in the equation of $g(x)$? Explain your reasoning.

+1 $\lim_{x \to -\infty} g(x) = \infty$, $\lim_{x \to \infty} g(x) = -\infty$ $\Big\}$ $\therefore g(x)$ is odd degree with $a < 0$ +1

c. How many points of inflection does the graph of $f(x)$ have? Give a reason for your answer.

- $f(x)$ has 2 pairs of consecutive extrema.
- Points of inflection occur about halfway between consecutive extrema.

$\therefore$ $f(x)$ has 2 points of inflection.

+1

+1

d. If d represents the constant term in the equation of $g(x)$, what can be concluded about the value of d? Explain your reasoning.

$g(x)$ crosses the x-axis at the origin.

$\therefore$ $g(x)$ has a factor of x.

$\therefore$ $g(x)$'s constant term $d=0$.

+1