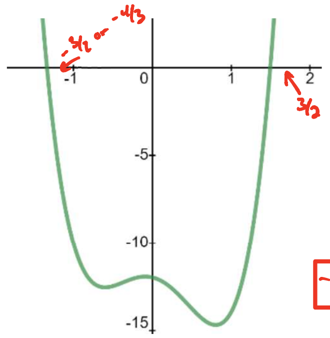
Homework 4.3

For exercises 1 – 4, list the possible rational roots of the given function. Then, find all roots, real and/or imaginary, of the function. Leave all answers in lowest terms and exact form.

1. $h(x) = 6x^4 - x^3 - 6x^2 - x - 12$



$PRR = \dfrac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm2, \pm3, \pm6}$

$HPRR = 3/2, \underbrace{-3/2 \text{ or } -4/3}$

$\boxed{3/2}$	6	-1	-6	-1	-12
	0	9	12	9	12
$\boxed{-4/3}$	6	8	6	8	$\lfloor 0$
	0	-8	0	-8	
	6	0	6	$\lfloor 0$	

$h(x) = (x - 3/2)(x + 4/3)(6x^2 + 6)$

$6x^2 + 6 = 0$

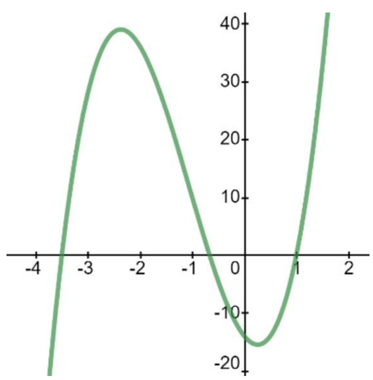
$6x^2 = -6$

$x^2 = -1$

$x = \pm i$

$\boxed{\text{Roots: } -4/3, 3/2, -i, i}$

2. $g(x) = 6x^3 + 19x^2 - 11x - 14$



$PRR = \dfrac{\pm1, \pm2, \pm7, \pm14}{\pm1, \pm2, \pm3, \pm6}$

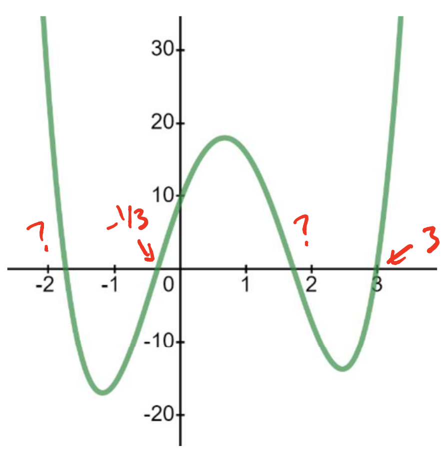
$HPRR = -\frac{7}{2}, -2/3, 1$

$\boxed{1}$	6	19	-11	-14
	0	6	25	14
$\boxed{-2/3}$	6	25	14	$\lfloor 0$
	0	-4	-14	
$\boxed{-7/2}$	6	21	$\lfloor 0$	
	0	-21		
	6	$\lfloor 0$		

$g(x) = (x-1)(x+2/3)(x+\frac{7}{2})6$

$\boxed{\text{Roots: } -\frac{7}{2}, -2/3, 1}$

3. $h(x) = 3x^4 - 8x^3 - 12x^2 + 24x + 9$



$PRR = \dfrac{\pm1, \pm3, \pm9}{\pm1, \pm3}$

$HPRR = -\frac{1}{3}, 3$

$\boxed{3}$	3	-8	-12	24	9
	0	9	3	-27	-9
$\boxed{-\frac{1}{3}}$	3	1	-9	-3	$\lfloor 0$
	0	-1	0	3	
	3	0	-9	$\lfloor 0$	

$h(x) = (x-3)(x+\frac{1}{3})(3x^2-9)$

$3x^2 - 9 = 0$

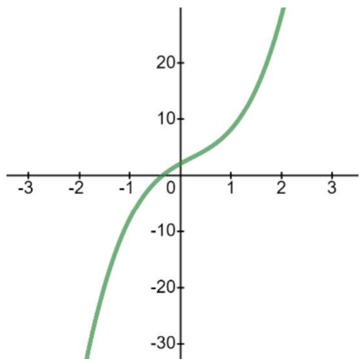
$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$\boxed{\text{Roots: } -\frac{1}{3}, 3, -\sqrt{3}, \sqrt{3}}$

4. $g(x) = 3x^3 - 2x^2 + 5x + 2$



$PRR = \frac{\pm 1, \pm 2}{\pm 1, \pm 3}$

$HPRR = -\frac{1}{3}$

$$\begin{array}{r|rrrr} \boxed{-\frac{1}{3}} & 3 & -2 & 5 & 2 \\ & 0 & -1 & 1 & -2 \\ \hline & 3 & -3 & 6 & \underline{0} \end{array}$$

$g(x) = (x + \frac{1}{3})(3x^2 - 3x + 6) = 0$

$g(x) = (x + \frac{1}{3})3(x^2 - x + 2)$

$x^2 - x + 2 = 0$

$Disc = b^2 - 4ac$

$= (-1)^2 - 4(1)(2)$

$= 1 - 8$

$Disc = -7$

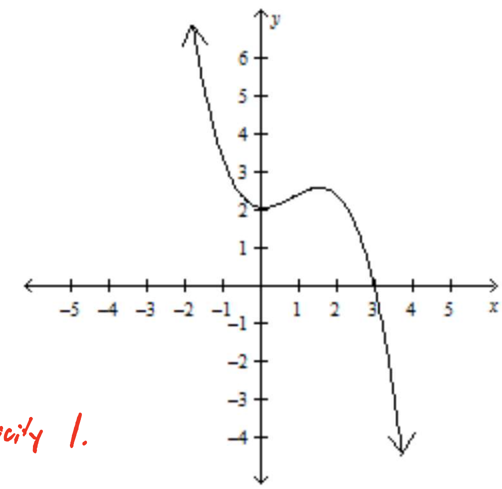
$x = \frac{-b \pm \sqrt{Disc}}{2a}$

$x = \frac{-(-1) \pm \sqrt{-7}}{2(1)}$

$x = \frac{1 \pm i\sqrt{7}}{2}$

Roots: $-\frac{1}{3}, \frac{1 - i\sqrt{7}}{2}, \frac{1 + i\sqrt{7}}{2}$

The graph of a quintic polynomial function, $p(x)$, is shown to the right. Use the graph to answer questions 5 – 8.



5. If a is the leading coefficient of the equation of $p(x)$, is $a < 0$ or is $a > 0$? Give a reason for your answer.

- $p(x)$ has ODD degree of 5
- $\lim_{x \to -\infty} p(x) = \infty$
- $\lim_{x \to \infty} p(x) = -\infty$

$\therefore a < 0.$

6. How many roots of $p(x)$ are imaginary? Give a reason for your answer.

By FTA, $p(x)$ has 5 roots b/c its quintic.

The graph of $p(x)$ has only 1 real root with multiplicity 1.

Thus the remaining 4 roots are imaginary.

7. If c is the constant term of the equation of $p(x)$, what is the value of c? Give reason for your answer.

The constant of an equation is the y-value of the y-int.

The graph of $p(x)$ has y-int of $(0, 2)$, $\therefore c = 2$

8. Is it possible that there are four sign changes in the equation of $p(x)$? Give a reason for your answer.

If $p(x)$ had 4 sign changes, Decartes' Rule of Signs says there will be 4, 2 or 0 positive roots.

But the graph of $p(x)$ only shows 1 positive root.

$\therefore$ $p(x)$ cannot have 4 sign changes.