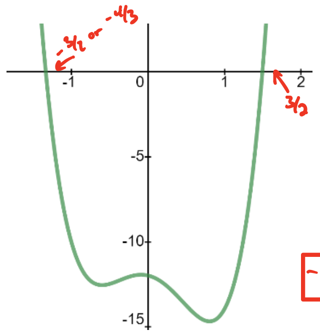
Homework 4.3

For exercises 1 – 4, list the possible rational roots of the given function. Then, find all roots, real and/or imaginary, of the function. Leave all answers in lowest terms and exact form.

1. $h(x) = 6x^4 - x^3 - 6x^2 - x - 12$



$PRR = \dfrac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 3, \pm 6}$

$HPRR = 3/2,\ \underbrace{-3/2 \text{ or } -4/3}$

$\boxed{\tfrac{3}{2}}$

6	-1	-6	-1	-12
0	9	12	9	12
6	8	6	8	$\boxed{0}$

$\boxed{-4/3}$

0	-8	0	-8
6	0	6	$\boxed{0}$

$h(x) = (x - 3/2)(x + 4/3)(6x^2 + 6)$

$6x^2 + 6 = 0$

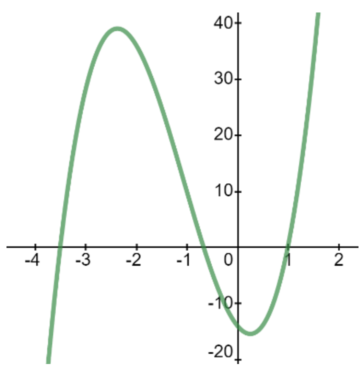
$6x^2 = -6$

$x^2 = -1$

$x = \pm i$

$\boxed{\text{Roots: } -4/3,\ 3/2,\ -i,\ i}$

2. $g(x) = 6x^3 + 19x^2 - 11x - 14$



$PRR = \dfrac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 2, \pm 3, \pm 6}$

$HPRR = -\tfrac{7}{2}, -2/3, 1$

$\boxed{1}$

6	19	-11	-14
0	6	25	14
6	25	14	$\boxed{0}$

$\boxed{-2/3}$

0	-4	-14
6	21	$\boxed{0}$

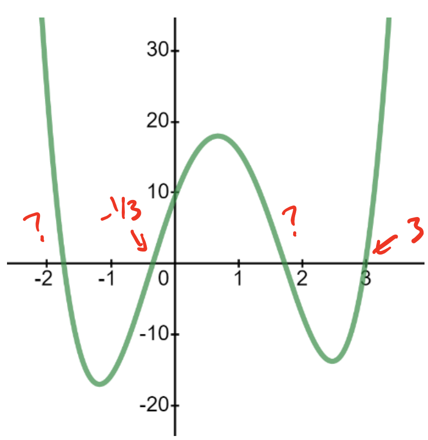
$\boxed{-7/2}$

0	-21
6	$\boxed{0}$

$g(x) = (x-1)(x+2/3)(x+\tfrac{7}{2})6$

$\boxed{\text{Roots: } -\tfrac{7}{2}, -2/3, 1}$

3. $h(x) = 3x^4 - 8x^3 - 12x^2 + 24x + 9$



$PRR = \dfrac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 3}$

$HPRR = -\tfrac{1}{3}, 3$

$\boxed{3}$

3	-8	-12	24	9
0	9	3	-27	-9
3	1	-9	-3	$\boxed{0}$

$\boxed{-\tfrac{1}{3}}$

0	-1	0	3
3	0	-9	$\boxed{0}$

$h(x) = (x-3)(x+\tfrac{1}{3})(3x^2-9)$

$3x^2 - 9 = 0$

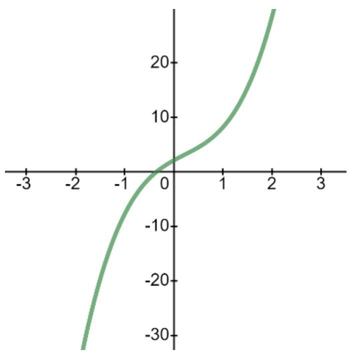
$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$\boxed{\text{Roots: } -\tfrac{1}{3}, 3, -\sqrt{3}, \sqrt{3}}$

4. $g(x) = 3x^3 - 2x^2 + 5x + 2$



$PRR = \frac{\pm 1, \pm 2}{\pm 1, \pm 3}$

$HPRR = -\frac{1}{3}$

$\boxed{-\frac{1}{3}}$

$$\begin{array}{cccc} 3 & -2 & 5 & 2 \\ 0 & -1 & 1 & -2 \\ \hline 3 & -3 & 6 & \underline{|0} \end{array}$$

$g(x) = (x+\frac{1}{3})(3x^2 - 3x + 6) = 0$

$g(x) = (x+\frac{1}{3})3(x^2 - x + 2)$

$x^2 - x + 2 = 0$

$Disc = b^2 - 4ac$
$= (-1)^2 - 4(1)(2)$
$= 1 - 8$
$Disc = -7$

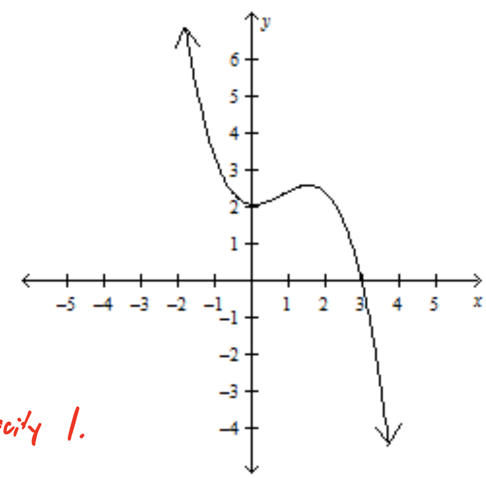
$x = \frac{-b \pm \sqrt{Disc}}{2a}$

$x = \frac{-(-1) \pm \sqrt{-7}}{2(1)}$

$x = \frac{1 \pm i\sqrt{7}}{2}$

Roots: $-\frac{1}{3}, \frac{1 - i\sqrt{7}}{2}, \frac{1 + i\sqrt{7}}{2}$

The graph of a quintic polynomial function, *p*(*x*), is shown to the right. Use the graph to answer questions 5 – 8.



5. If *a* is the leading coefficient of the equation of *p*(*x*), is $a < 0$ or is $a > 0$? Give a reason for your answer.

$\lim_{x \to \infty} p(x) = -\infty$

$\therefore a < 0.$

6. How many roots of *p*(*x*) are imaginary? Give a reason for your answer.

By FTA, p(x) has 5 roots b/c its quintic.

The graph of p(x) has only 1 real root with multiplicity 1.

Thus the remaining 4 roots are imaginary.

7. If *c* is the constant term of the equation of *p*(*x*), what is the value of *c*? Give reason for your answer.

The constant of an equation is the y-value of the y-int.

The graph of p(x) has y-int of (0,2), $\therefore c = 2$

8. Is it possible that there are four sign changes in the equation of *p*(*x*)? Give a reason for your answer.

If p(x) had 4 sign changes, Decartes' Rule of Signs says there will be 4, 2 or 0 positive roots.

But the graph of p(x) only shows 1 positive root.

$\therefore$ p(x) cannot have 4 sign changes.